You are designing a closed-loop control for a DC servo position control mechanism to be used in a process with desired settling times of 0.5 seconds. Catalogue the following pole positions into GOOD, BAD, BORDERLINE, according to their suitability.

A=-3, B=-0.5, C=-10, D=0.5, E=4, F=8, G=0.2+3j, H=-2+2j, I=2+6j, J=-10+5j, K=-2+4j, L=2+2j, M=-5+j

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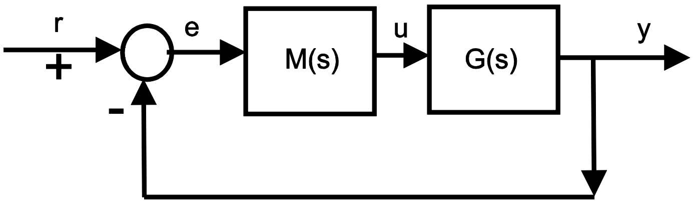
Use the Routh criteria to determine whether the following polynomials have any RHP roots. Give your reasoning.

$s^2 + 2s - 0.1 \qquad\qquad s^3 + s^2 + 2s + 0.1$

Define the closed-loop pole polynomial for the following pairs, assuming the loop structure given here.



$$\begin{Bmatrix} G = \dfrac{1}{s^2+3s+2} \\ M(s) = \dfrac{0.4}{s}K \end{Bmatrix}; \quad \begin{Bmatrix} G = \dfrac{1}{s^3+3s^2+3s+1} \\ M(s) = \dfrac{0.2(s+3)}{(s+4)}K \end{Bmatrix}; \quad \begin{Bmatrix} G = \dfrac{1}{s^3+6s^2+11s+6} \\ M(s) = \dfrac{0.2}{s}K \end{Bmatrix}$$

For what values of K will these systems be closed-loop stable?

Sketch the root-loci for the following system (G(s), K(s) are connected with unity negative feedback as in question 2) and mark on your figure where you are likely to want the dominant closed-loop poles to be. Explain your working clearly.

$$G(s) = \frac{4(s+0.5)}{s(s-1)(s+3)}; \quad K(s) = 2\frac{s+2}{s+6}$$

Use MATLAB to extract the closed-loop poles for the following compensator/system pair given that K has values of 0, 0.1, 0.3, 0.5, 0.8, 1, 2, 5.

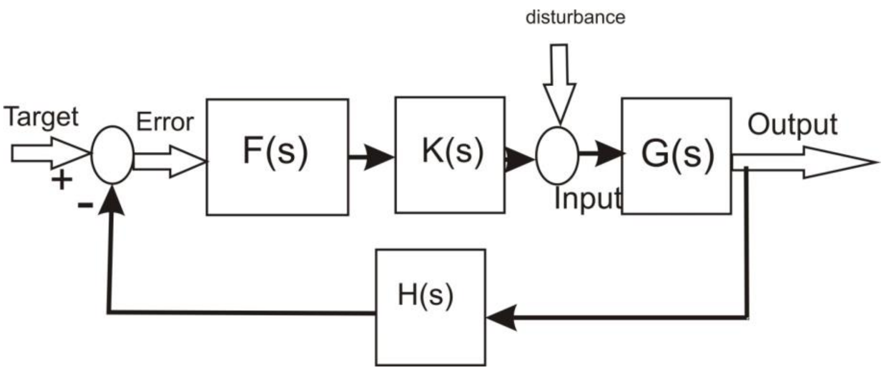
$$G(s) = \frac{4(s+0.5)}{s(s-1)(s+3)}; \quad M(s) = 2K\frac{s+2}{s+6}$$

Also use MATLAB to plot the root-loci for this.

Give an explanation or definition for root-loci and hence, using examples of your choice, show from first principles how root-loci can be sketched.

Derive the 5 simple rules for sketching root-loci. What are the key identities that underpin these and where do they come from? Would these rules change if a loop contained numerous blocks, included a sensor in the return path? How would these rules change for positive feedback?

Sketch the root-loci for the following loop using the values given:



$$G(s)=\frac{4(s+0.5)}{s(s-1)(s+3)};\quad K(s)=2\frac{s+2}{s+6}$$

$$F(s)=K\frac{s+0.1}{s+0.02};\quad H(s)=0.2\frac{s+5}{s+1}$$

Sketch the root-loci for the following systems. Propose values for compensator gain K.

$$G(s)=\frac{3K(s+9)}{s(s+2)(s+4)}$$

$$\left\{\begin{matrix} G=\dfrac{1}{s^3+6s^2+11s+6} \\ K(s)=\dfrac{K}{s}\end{matrix}\right\}\quad \left\{\begin{matrix} G=\dfrac{1}{s^2+3s+2} \\ K(s)=K\end{matrix}\right\}\quad \left\{\begin{matrix} G=\dfrac{1}{s^2+3s+2} \\ K(s)=\dfrac{K}{s}\end{matrix}\right\}\quad \left\{\begin{matrix} G=\dfrac{1}{s^3+3s^2+3s+1} \\ K(s)=\dfrac{0.2(s+3)}{(s+4)}K\end{matrix}\right\}$$

$$\left\{\begin{matrix} G=\dfrac{1}{s^3+6s^2+11s+6} \\ K(s)=\dfrac{(s+6)}{(s+4)}K\end{matrix}\right\}\quad \left\{\begin{matrix} G=\dfrac{1}{s^3+6s^2+11s+6} \\ K(s)=\dfrac{0.2}{s}K\end{matrix}\right\}$$

$$\left\{\begin{matrix} G=\dfrac{1}{s^2+2s+3} \\ K\end{matrix}\right\};\left\{\begin{matrix} G=\dfrac{1}{s^3+9s^2+23s+15} \\ K\end{matrix}\right\};\left\{\begin{matrix} G=\dfrac{(s+3)}{(s+1)(s+2)(s+4)} \\ K\end{matrix}\right\}$$

$$\left\{\begin{matrix} G=\dfrac{s+3}{s^2+2s+1} \\ K(s)=\dfrac{K}{s}\end{matrix}\right\};\left\{\begin{matrix} G=\dfrac{(s+0.5)}{s^3+9s^2+23s+15} \\ K(s)=K\dfrac{(s+4)}{s}\end{matrix}\right\};\left\{\begin{matrix} G=\dfrac{(s+2)}{(s+10)(s^3+4s^2+3s)} \\ K\end{matrix}\right\}$$

$$s^3+9s^2+23s+15=(s+1)(s+3)(s+5)$$

Use MATLAB to check your answers and in particular to check whether your proposed values for K do indeed work as expected.

Old exam question: A control system is given by transfer functions below. Sketch the root-loci showing all your working. Estimate the value of K such that the root-loci cuts the imaginary axis. What are the corresponding closed-loop poles? [Check your answers with MATLAB]

$$GK=\frac{K(s+3)}{s(s+1)(s+2)(s+4)}\qquad GK=\frac{K}{s(s+4)(s^2+8s+32)}\qquad G=\frac{K}{s^3+9s^2+23s+15};\ G=\frac{4K}{s(s+2)(s+3)};$$

Take a simple system, place in a loop simple negative feedback and compensator K and estimate the gain K to get the best compromise between speed of response and input activity. Use sisotool for this exercise.

$$G=\frac{1}{s(s+1)^2} \qquad G=\frac{1}{s(s+2)(s+3)} \qquad G=\frac{(s+4)}{s(s+2)(s+3)}$$

Assuming unity negative feedback and compensator K, use sisotool to find K to set the dominant poles for systems below as having a damping ratio of 0.7. Observe the closed-loop responses and decide if this is a good tuning.

$$G=\frac{4}{(s+2)(s+1)};\quad G=\frac{1}{s^3+9s^2+23s+15}$$
$$G=\frac{s+3}{(s+2)(s+1)(s+4)};\quad G=\frac{s+4}{s(s+2)(s+3)}$$

What is the impact on root-loci of adding a lag compensator? Illustrate with an example.
What is the impact on root-loci of adding a lead compensator? Illustrate with an example.

Suggest suitable values for the poles and zeros of lead and lag compensators in the light of your observations. What have you observed about lead and lag compensators? What advantages do they each bring? When would you use them and why?

Investigate the root-loci and corresponding closed-loop time responses for the following system triples and hence determine which are good/bad compensator designs. Also compare steady-state offsets for each case.

$$G=\frac{4}{(s+2)(s+1)};\quad K_{lag}=K\frac{s+0.2}{s+0.05};\quad K_{lead}=K\frac{s+1}{s+3}$$
$$G=\frac{1}{s^3+9s^2+23s+15};\quad K_{lag}=K\frac{s+0.2}{s+0.05};\quad K_{lead}=K\frac{s+1}{s+3}$$
$$G=\frac{s+3}{(s+2)(s+1)(s+4)};\quad K_{lag}=K\frac{s+0.2}{s+0.05};\quad K_{lead}=K\frac{s+1}{s+3}$$
$$G=\frac{s+4}{s(s+2)(s+3)};\quad K_{lag}=K\frac{s+0.2}{s+0.05};\quad K_{lead}=K\frac{s+1}{s+3}$$

Use root-loci to show why one of the following systems, when connected with unity negative feedback, is always closed-loop unstable and the other is always closed-loop stable, for all positive values of K. What would happen if you deployed positive feedback?

$$G_1=\frac{s+1}{s^2(s+2)};\quad G_2=\frac{s+2}{s^2(s+1)}$$

Why for examples G_1, G_3 is there a minimum and maximum value of K to get optimum behaviour and for G_2 a maximum only?

$$G_1=\frac{(s+3)}{s(s+1)};\quad G_2=\frac{s+1}{(s-1)s};\quad G_3=\frac{(s+1)}{s(s+0.5)(s+10)}$$