The full definition of a Nyquist diagram is the mapping of $G(s)$ while $s$ describes the D-contour. A system is closed-loop stable ($n_c=$number RHP closed-loop poles) if and only if the number of counter-clockwise encirclements of -1 point matches number $n_o$ of open-loop RHP poles.

**Alternatively, apply** $n_q=n_c-n_o$ **directly or** $n_c=n_q+n_o$

### Apply Nyquist stability criteria to numerous examples

**Case 1:**

$$G = \frac{5}{s+4};$$

$$n_o = 0$$
$$n_q = 0$$

$\Rightarrow n_c = 0$

CLOSED-LOOP STABLE
(For all +ve $K$)

![Nyquist plot](image1)

**Case 2:**

$$G = \frac{3}{(s+2)(s+1)};$$

$$n_o = 0$$
$$n_q = 0$$

$\Rightarrow n_c = 0$

CLOSED-LOOP STABLE
(For all +ve $K$)

![Nyquist plot](image2)
\[ G = \frac{5}{(s+1)^3}; \quad K = 1 \]
\[ n_o = 0 \]
\[ n_q = 0 \]
\[ \Rightarrow \quad n_c = 0 \]

Clearly stable for current \( K \).

However, \( K > 8/5 \) implies that \( n_q = 2 \) and hence one will have \( n_c = 2 \), that is closed-loop instability.

\[ G = \frac{s + 0.1}{(s + 0.2)(s - 1)}; \quad K = 1 \]
\[ n_o = 1 \]
\[ n_q = 0 \]
\[ \Rightarrow \quad n_c = 1 \]

Clearly **unstable** for current \( K \) (a closed-loop RHP pole).

However, \( K > 2 \) implies that \( n_q = -1 \) and hence one will have \( n_c = 0 \), that is closed-loop stability.

\[ G = \frac{40(s + 2)}{(s + 10)(s + 4)(s - 1)}; \]
\[ n_o = 1 \]
\[ n_q = -1 \]
\[ \Rightarrow \quad n_c = 0 \]

Clearly stable for current \( K = 1 \). However, \( K < 0.5 \) will remove the counter clockwise encirclement and hence closed-loop instability would follow.

**REMARKS** The result is based on two things which students must be skilled at:

1. Counting open-loop RHP poles – a common mistake is to forget the distinction between LHP and RHP poles. Only count open-loop RHP poles. This is \( n_o \).
2. Sketch the Nyquist diagram of the loop transfer function and count encirclements of the -1 point. This is \( n_q \).

Determine the number of RHP closed-loop poles from \( n_c = n_q + n_o \).

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