Modelling and control summaries

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Block diagrams 1 – the transfer function

KEY SKILLS: Before using block diagrams, students must have clarity on what constitutes a transfer function and how this differs from a Laplace transform.

Laplace transforms represent signals. **Transfer functions represent systems.**

Concept of a system with an input U(s) and an output Y(s)

Imagine you are only interested in the speed of a car.

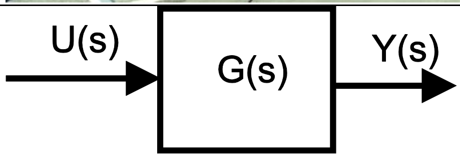
- The input is clearly defined as the throttle position.
- The output is defined as the car speed.

The speed depends upon two things:

- The choice of input (or throttle)
- The car dynamics (that is the system).

Students will see two different concepts affect behaviours: (i) a signal – here the throttle and (ii) a system – here the car.

A block diagram represents this dependence conceptually by showing the input as an arrow coming into a block which represents the system (here G(s)) and the output as an arrow coming out of the system.

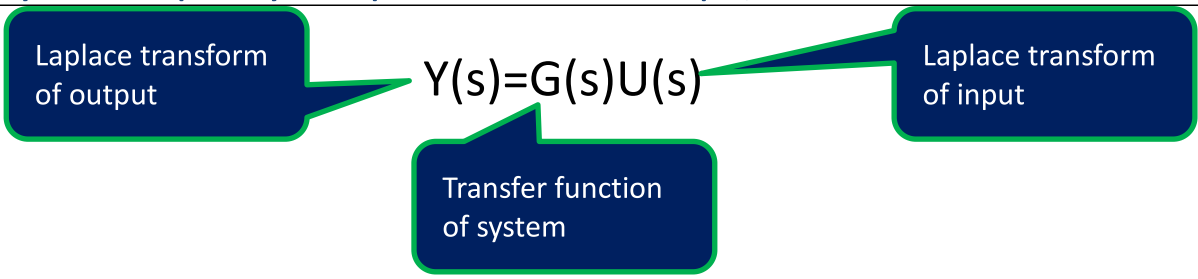


EXPLICIT DEPENDENCE OF THE OUTPUT ON THE INPUT

For a linear system represented by a differential equation, the relationship between Y(s) and U(s) can be determined explicitly using simple algebra. Take Laplace transforms of every term noting that initial conditions are always ignored in block diagram algebra.

1st order example G(s) is a transfer function as it gives the dependency between U(s) and Y(s).	$a\frac{dy}{dt}+by=ku \quad\Rightarrow\quad (as+b)Y(s)=kU(s)$ $Y(s)=\underbrace{\left[\frac{k}{as+b}\right]}_{G(s)}U(s)=G(s)U(s)$
2nd order example G(s) is a transfer function as it gives the dependency between U(s) and Y(s).	$a\frac{d^2y}{dt^2}+b\frac{dy}{dt}+cy=ku \quad\Rightarrow\quad (as^2+bs+c)Y(s)=kU(s)$ $Y(s)=\underbrace{\left[\frac{k}{as^2+bs+c}\right]}_{G(s)}U(s)=G(s)U(s)$

SUMMARY OBSERVATIONS - the Laplace transform of the output is given by the transfer function of the system multiplied by the Laplace transform of the input, that is:



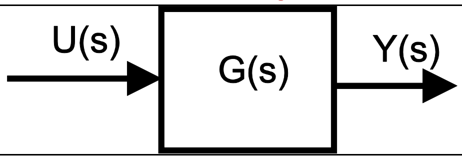
The coefficients of G(s) are made up from the parameters of the system model (differential equation) and hence this represents the system dynamics.

INTERPRETING BLOCK DIAGRAMS

1. Lines represent signals and blocks represent systems.
2. An arrow (or line) into a block represents a system input. (Arrows are not compulsory but clarify the direction of flow from input to output).
3. An arrow (or line) out of a block represents a system output.
4. Implicit that the Laplace transform of the output is the transfer function multiplied by the Laplace transform of the input.

The following two representations are equivalent

Y(s)=G(s)U(s)



REMARK on behaviours:

1. The output Y(s) comprises both U(s) and G(s).
2. The output behaviour comprises the behaviours in the input and the behaviours of the system dynamics.
3. This is exactly what we would expect, that is, the system or inherent dynamics must be present but we can add onto these forced dynamics by our choice of input signal.

CONVERGENCE and ASYMPTOTIC BEHAVIOURS OF SYSTEM OUTPUTS:

Most real systems have stable dynamics, that is, with no input, the system will decay to zero. Consequently, if we need a system to settle at a non-zero point, this behaviour must be supplied by the input signal: (i) a step input to give a steady non-zero output; (ii) a ramp input to give a ramp output; (iii) a sinusoidal input to give a sinusoidal output; and so on!

HIGH ORDER EXAMPLE

$$a\frac{d^n y}{dt^n}+b\frac{d^{n-1}y}{dt^{n-1}}+\cdots f\frac{dy}{dt}+gy=k\frac{d^2u}{dt^2}+m\frac{du}{dt}+nu$$

$$\Rightarrow \quad (as^n+bs^{n-1}+\cdots+fs+g)Y(s)=(ks^2+ms+n)U(s) \quad \Rightarrow \quad Y(s)=\underbrace{\left[\frac{ks^2+ms+n}{as^n+bs^{n-1}+\cdots+fs+g}\right]}_{G(s)}U(s)$$