Modelling and control summaries

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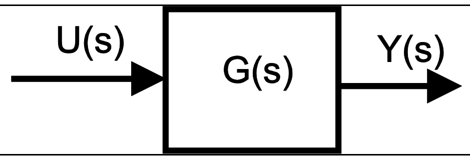
Block diagrams 2 – systems in series

INTERPRETING BLOCK DIAGRAMS: Lines represent signals and blocks represent systems. An arrow (or line) into a block represents a system input. An arrow (or line) out of a block represents a system output. Laplace transform of the output is the transfer function multiplied by the Laplace transform of the input. HENCE

Y(s)=G(s)U(s) is equivalent to



Systems in series

Consider the context where the output from system 1 is the input to system 2. This can be represented by two differential equations as illustrated here.

- System 1 has input u and output y.
- System 2 has input y and output x.

System 1: $$a\frac{dy}{dt}+by=ku \quad and \quad c\frac{dx}{dt}+ex=my$$:System 2

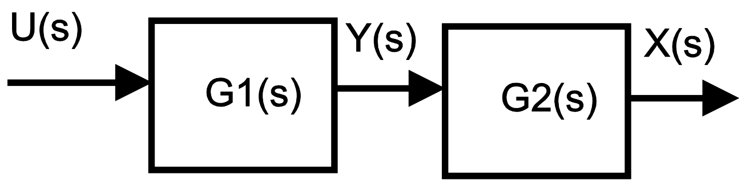
The overall system can now be modelled by using Laplace transforms as in the previous note.

$$Y(s)=\underbrace{\left[\frac{k}{as+b}\right]}_{G_1(s)}U(s)=G_1(s)U(s); \quad X(s)=\underbrace{\left[\frac{m}{cs+e}\right]}_{G_2(s)}Y(s)=G_2(s)Y(s)$$

Combining these two equations and eliminating Y(s), one finds:

$$X(s)=G_2(s)Y(s)=\underbrace{[G_2(s)G_1(s)]}_{G(s)}U(s)=G(s)U(s)$$

A block diagram is a good way of represents this linkage using arrows to represent the change of Y(s) from an output of system 1 to an input of system 2.



$$X(s)=G_2(s)G_1(s)U(s)$$

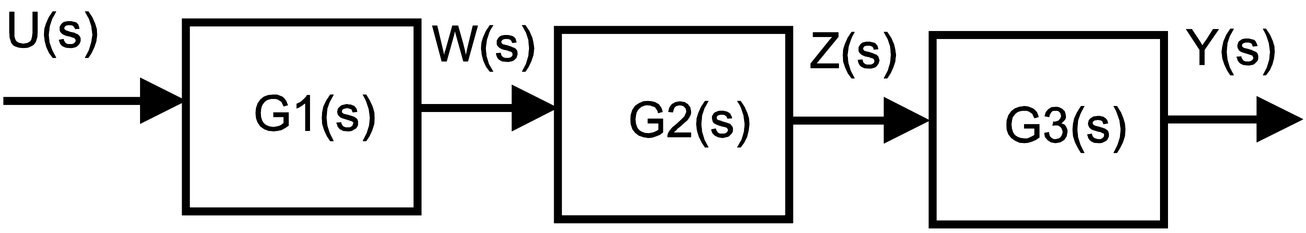
<u>REMARK:</u> This concept is easily extended to multiple systems in series and in practice users sketch the block diagram first and then write down the transfer function relationships by inspection.

EXAMPLE OF 3 SYSTEMS IN SERIES AND THE BLOCK DIAGRAM REPRESENTATION

$$\frac{dw}{dt}+4w=2u; \quad \frac{dz}{dt}+z=w; \quad \frac{dy}{dt}+3y=4z;$$

$$W(s)=\underbrace{\left[\frac{2}{s+4}\right]}_{G_1(s)}U(s); \quad Z(s)=\underbrace{\left[\frac{1}{s+1}\right]}_{G_2(s)}W(s); \quad Y(s)=\underbrace{\left[\frac{4}{s+3}\right]}_{G_3(s)}Z(s)$$

$$\Rightarrow \quad Y(s)=G_3(s)G_2(s)G_1(s)U(s)$$



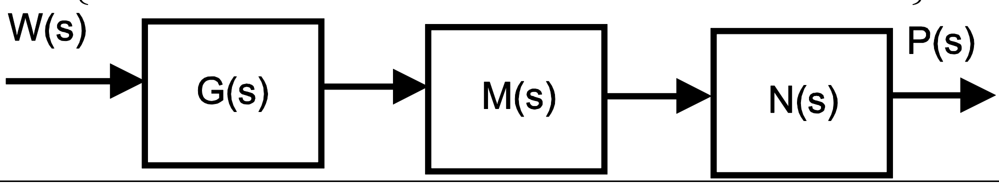
QUESTION 1: Find a transfer function representation for the series system represented by the following two ODEs.

$$u=4\frac{dx}{dt}+2x; \quad 2x=\frac{dy}{dt}+y$$

$$\left\{X(s)=\underbrace{\left[\frac{1}{4s+2}\right]}_{G(s)}U(s); \quad Y(s)=\underbrace{\left[\frac{2}{s+1}\right]}_{M(s)}X(s)\right\} \Rightarrow \quad Y(s)=\left[\frac{2}{4s^2+6s+2}\right]U(s)$$

QUESTION 2: Given the following transfer functions and block diagram, find a relationship between W(s) and P(s).

$$\left\{G(s)=\frac{1}{4s+2}; \quad M(s)=\frac{2}{s+1}; \quad N(s)=\frac{0.42}{s^2+s+1}\right\}$$



Using the definition of block diagram algebra, by inspection

P(s)=[N(s)M(s)G(s)]W(s) and hence

$$P(s)=\left(\frac{0.42}{s^2+s+1}\right)\left(\frac{2}{s+1}\right)\left(\frac{1}{4s+2}\right)W(s)$$