Modelling and control summaries

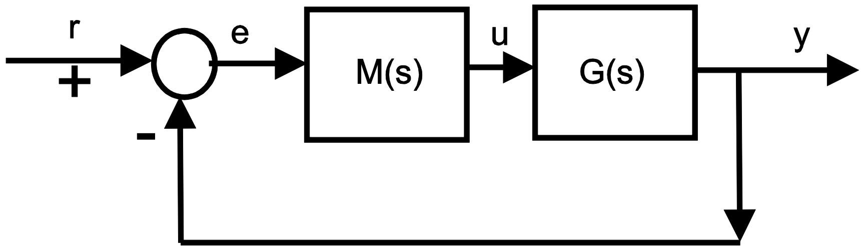
by Anthony Rossiter

Block diagrams 5 – short cuts

Once students are confident determining closed-loop transfer functions long hand using the technique of block diagrams 4, then they can derive some short cut methods to same time and give derivations for complex arrangements almost by inspection.

Key observations: Students should be confident in the following derivations before continuing.

$$y = \frac{GM}{1+GM} r$$
$$u = \frac{M}{1+GM} r$$
$$e = \frac{1}{1+GM} r$$



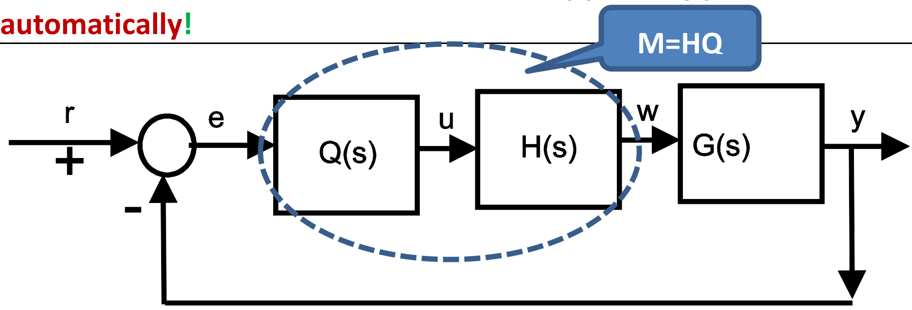
KEY POINTS:

1. The denominator is always the same, that is 1+GM where GM is all the transfer functions in the loop path.
2. The numerator comprises only the transfer functions between the summing junction and the signal in question. (e.g. GM for signal y but only M for signal u)

Add more terms in the forward path

The trick here is to recognise that the diagram below can be represented as equivalent to the case above by combining two of the transfer functions. Here combine Q(s) and H(s) so M=HQ. Then the same results follow automatically!

$$y = \frac{GM}{1+GM} r$$
$$w = \frac{M}{1+GM} r$$
$$e = \frac{1}{1+GM} r$$

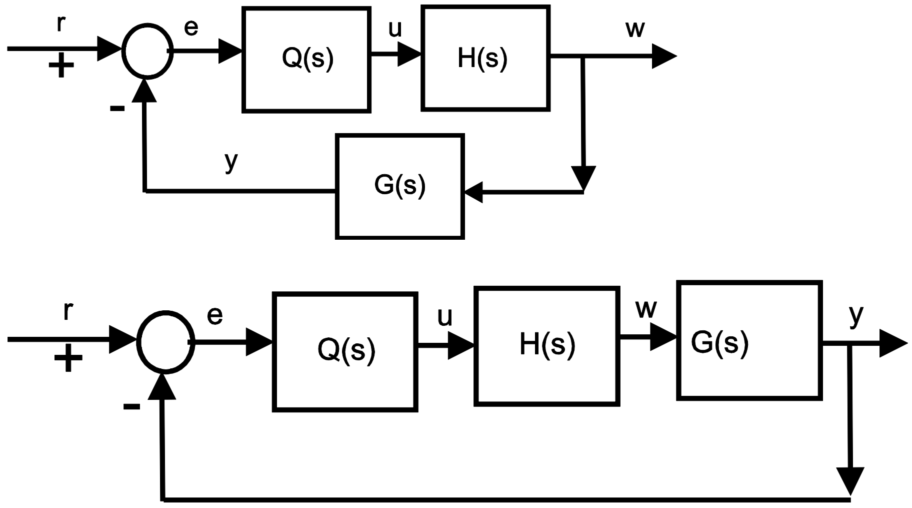


KEY POINTS:

1. The denominator is always the same, that is 1+GM=1+GHQ where GHQ is all the transfer functions in the loop path.
2. The numerator comprises only the transfer functions between the summing junction and the signal in question. (e.g. GHQ for signal y but only HQ=M for signal w).
3. One could use the same technique to find u by combining H(s) and G(s).

Change to the relationships with a sensor G(s) in the feedback path

The following diagram appears to be quite different, but actually it is exactly equivalent to the one beneath and hence again one can use the results of example 1 by simple combining blocks as desired.



$$
\begin{aligned}
y &= \frac{GHQ}{1+GHQ} r \\
w &= \frac{HQ}{1+GM} r \\
e &= \frac{1}{1+GM} r \\
u &= \frac{Q}{1+GMQ} r
\end{aligned}
$$

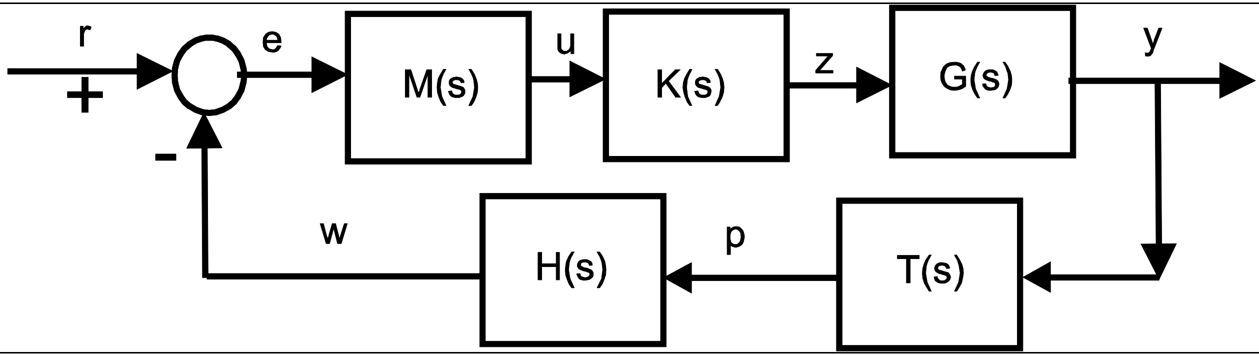
CHALLENGE: Make sure you can derive and use the following

The basic rule for finding a transfer function between a loop input r and a signal within the loop is the following formulae (assumes summing junction deploys negative feedback).

Closed-loop TF = [Forward path]/[1 + loop TF]

Loop TF is the product of all the transfer functions in the loop

Challenge question: Confirm the following results



$$
e = \frac{1}{1+HTGKM} r; \quad u = \frac{M}{1+HTGKM} r;
$$

$$
z = \frac{KM}{1+HTGKM} r; \quad y = \frac{GKM}{1+HTGKM} r;
$$