Modelling and control summaries

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Block diagrams 7 – nested loops

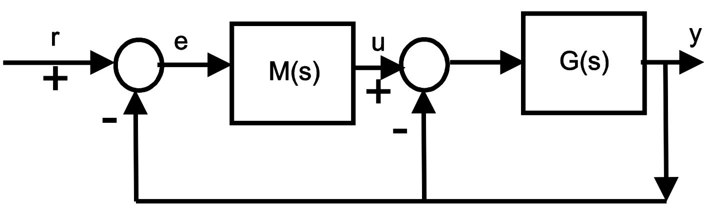
Real systems often have nested loops

While the textbooks have number of clever mechanisms for simplifying nested loops which could be useful for complicated cases, these notes focus on the more straightforward examples for which simple techniques are still effective and easy to use.

Key principles adopted in this note

1. A use of shortcut methods for finding transfer functions between loop inputs and signals in the loop.
2. Identify which signal is of interest and then group together any transfer functions where this does not affect the required computation.
3. Remember, you can always resort to long hand methods (simultaneous equations based on equations around each block and summing junction) to test your answers/understanding.

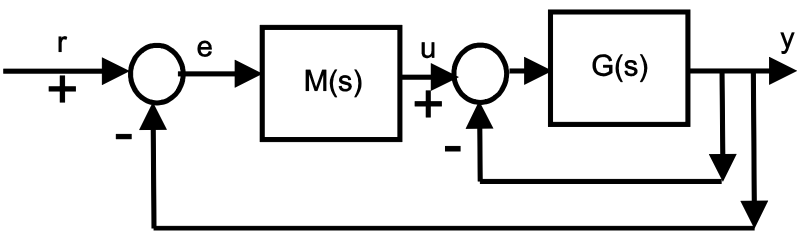
EXAMPLE 1

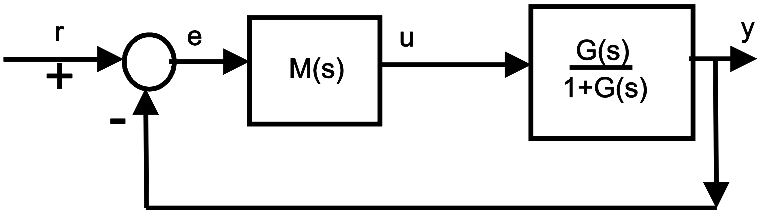


Longhand method

$$\left.\begin{array}{l} e = r - y \\ u = Me \\ f = u - y \\ y = Gf \end{array}\right\} \Rightarrow \begin{array}{l} u = M(r-y) \\ f = M(r-y) - y \\ y = G(M(r-y) - y) \end{array}$$

$$\Rightarrow \quad y = \frac{GM}{1+GM+G} r$$



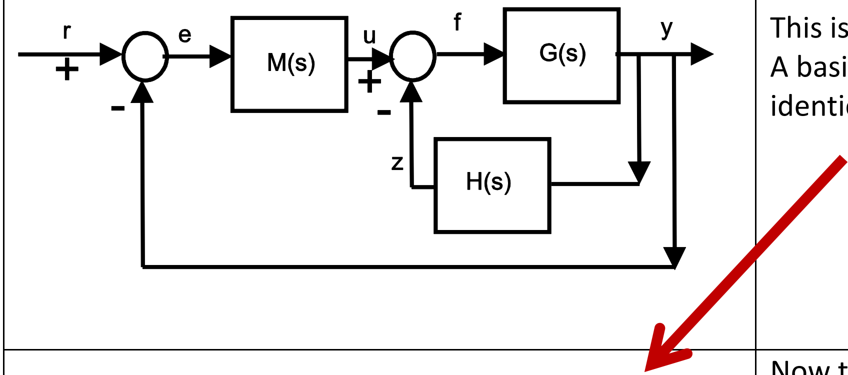


Rearrange diagram twice to expose the structure in the nested loops.

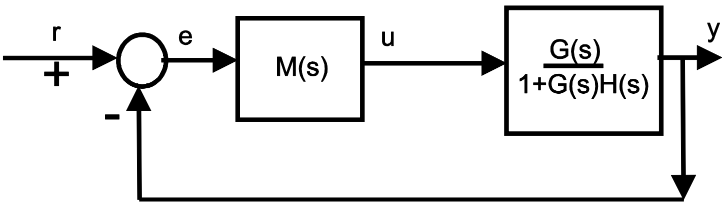
Now the loop transfer functions can be derived by inspection, for example:

$$u = \frac{M}{1 + M\dfrac{G}{1+G}} r = \frac{(1+G)M}{1+G+GM} r$$

EXAMPLE 2



This is in fact quite similar to example 1. A basic rearrangement gives a near identical structure change as shown here.



Now the loop transfer functions can be derived by inspection, for example:

$$y=\frac{M\dfrac{G}{1+GH}}{1+M\dfrac{G}{1+GH}}r=\frac{GM}{1+GH+GM}r$$

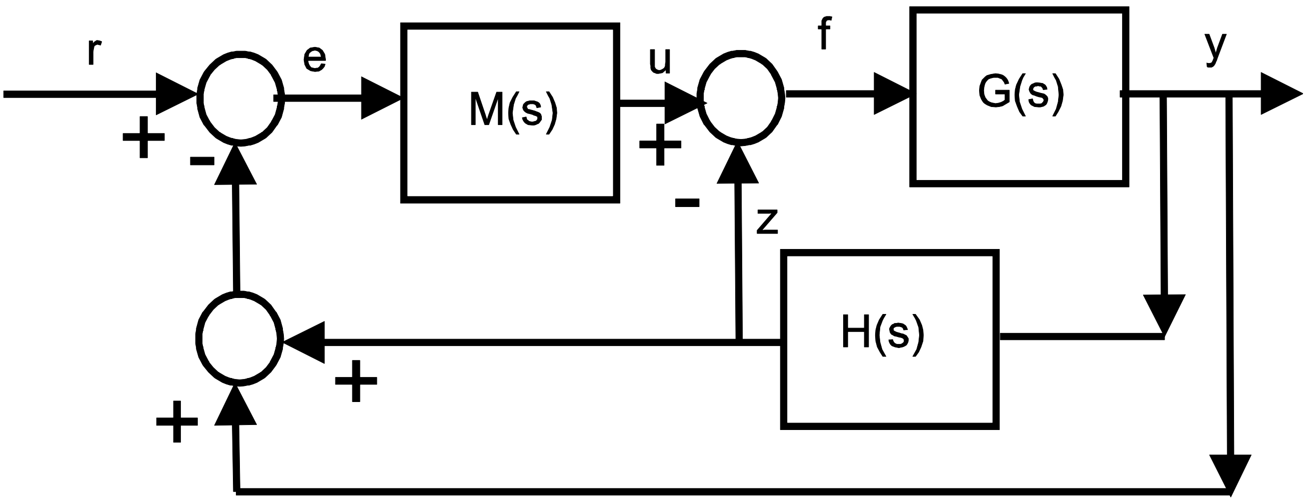
A more challenging question would be to determine the signal f which is inside the combined block. However, due to linearity this can be down in two simple steps both of which use short cut methods:

1. Determine signal u.
2. Determine dependence of f on u.
3. Combine these two together.

$$u=\frac{M}{1+M\dfrac{G}{1+GH}}r; \quad f=\frac{1}{1+GH}u$$

$$f=\frac{1}{1+GH}\frac{M}{1+M\dfrac{G}{1+GH}}r=\frac{M}{1+GH+GM}r$$

EXAMPLE 3: Try this by yourself



HINTS: $u=\dfrac{M}{1+MH\dfrac{G}{1+GH}}r; \quad f=\dfrac{1}{1+GH}u; \quad y=\dfrac{M\dfrac{G}{1+GH}}{1+MH\dfrac{G}{1+GH}}r;$