Modelling and control summaries

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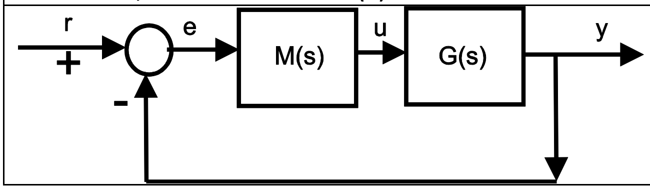
Intro. to feedback 5 – impact of feedback

FEEDBACK INVOLVES MEASUREMENT, DECISION MAKING BASED ON THE MEASUREMENT AND INPUT ADJUSTMENT. This changes behaviour. Here we consider the impact of proportional feedback, that is where M(s) is chosen to be a constant K.



CLOSED- LOOP (M(s)=K)

$$Y(s)=\frac{G(s)K}{1+G(s)K}R(s)=G_c(s)R(s)$$

Analysing performance: A systematic analysis of the impact of feedback and changes in feedback requires some objective measures of performance. Typical measures adopted are:

1. Stability. (Is the output convergent, that is, are all poles of G_c in the LHP?)
2. Speed of response/settling time. (Taken to be about 3 times dominant time constant).
3. Closed-loop gain/offset. (Does the output reach the target and if not, how big is offset?)
4. Shape of response. (Are their oscillations and overshoots and if so, how big?)

We will use only the first 3 criteria on some simple case studies as none of these oscillate.

LEVEL CONTROL OF A TANK

Process model (for given parameters)

$$A\frac{dh}{dt}+R\rho gh=f \quad\Rightarrow\quad h(s)=\frac{0.02}{s+0.02}f(s)$$

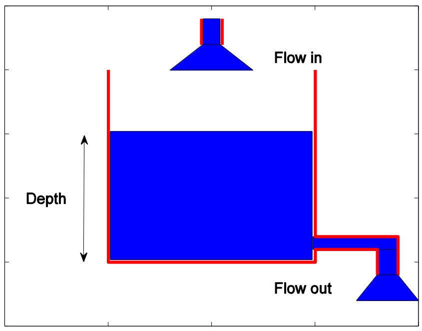
Proportional control law

f=K(r-h)

Closed-loop transfer function

$$G_c=\frac{G(s)K}{1+G(s)K}=\frac{0.02K}{s+0.02+0.02k};$$

$$offset=\frac{1}{1+G(0)K}=\frac{1}{1+K};\quad gain=\frac{K}{1+K}$$



1. POLE=-0.02(1+K) in LHP and stable if K>0. Gets further into LHP as K increases.
2. Time constant =1/(0.02(1+K)). Gets faster as K increases, that is T reduces.
3. Offset reduces as K increases but remains noticeably bigger than zero. Gain increases with K but always remains less than one.

SUMMARY: Changing the feedback gain changes both the time constant/pole and the steady-state gain/offset. However, with purely proportional gain, the closed-loop system does not deliver the system to the specified target level – there is a significant offset unless K is very large which cannot be implemented as f=K(r-h), that is, this implies unrealistic flows.

TEMPERATURE CONTROL IN A HOUSE

Process model (for given parameters): $C\frac{dt}{dt}+kT=W \quad \Rightarrow \quad T(s)=\frac{1}{Cs+k}W(s)$

Proportional control law: *W=K(r-T)*

Closed-loop transfer function (C=400000,k=1000)

$$G_c=\frac{G(s)K}{1+G(s)K}=\frac{0.001K}{4000s+1+0.001K}; \quad offset=\frac{1}{1+G(0)K}=\frac{1}{1+0.001K}; \quad gain=\frac{0.001K}{1+0.001K}$$

1. POLE=-(1+0.001K)/4000 in LHP and stable if K>0. Gets further into LHP as K increases.
2. Time constant =4000/(1+0.001K). Gets faster as K increases, that is T reduces.
3. Offset reduces as K increases but remains noticeably bigger than zero. Gain increases with K but always remains less than one.

SUMMARY: Changing the feedback gain changes both the time constant/pole and the steady-state gain/offset. However, with purely proportional gain, the closed-loop system does not deliver the system to the specified target level – there is a significant offset unless K is very large which cannot be implemented as W=K(r-T), that is, this implies an unrealistically large heat supply.

CRUISE CONTROL FOR A MOTOR VEHICLE

Process model (for mass=800kg, friction=200, typical force =4800N and f=4800u):

$$m\frac{dV}{dt}+bv=f \quad \Rightarrow \quad V(s)=\frac{1}{Ms+b}f(s)=\frac{4800}{800s+200}u(s)=\frac{24}{4s+1}u(s)$$

Proportional control law: *u=K(r-v)*

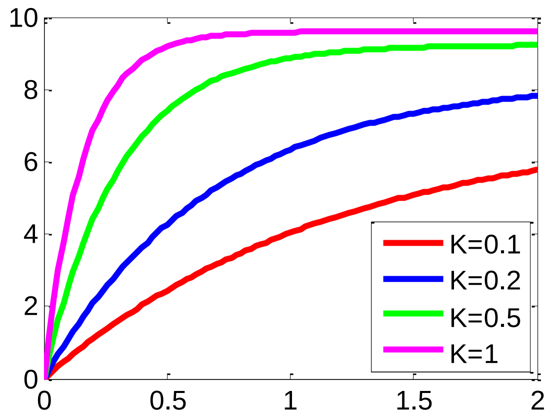
Closed-loop transfer function

$$G_c=\frac{G(s)K}{1+G(s)K}=\frac{24K}{4s+1+24K}; \quad offset=\frac{1}{1+24K}; \quad gain=\frac{24K}{1+24K}$$

1. POLE=- (1+24K)/4 in LHP and stable if K>0. Gets further into LHP as K increases.
2. Time constant =4/(1+24K). Gets faster as K increases, that is T reduces.
3. Offset reduces as K increases but remains noticeably bigger than zero. Gain increases with K but always remains less than one.

The figure shows closed-loop responses a speed demand of 10m/s. It is clear that the dynamics, gains and offsets are very different as K is changed.

SUMMARY: Changing the feedback gain changes both the time constant/pole and the steady-state gain/offset. However, with purely proportional gain, the closed-loop system does not deliver the system to the specified target level – there is a significant offset unless K is very large which cannot be implemented as u=K(r-v) and u is limited to values in the region of 1. If r=10, as here, then max(u)=10K, which implies max(f)=48000K! **You will note these accelerations are not achievable with a real car!**



SUMMARY: Changing the proportional feedback has a clear and obvious affect on closed-loop behaviour. In general larger values give faster responses and smaller offset, but nevertheless cannot remove offset entirely. Large values may not be implementable so the strategy of proportional feedback is severely limited.