Modelling and control summaries

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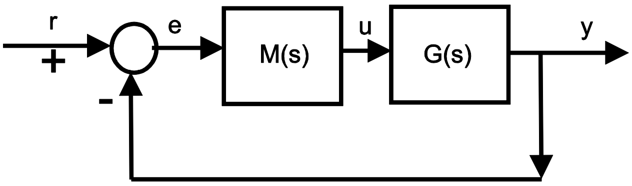
Intro. to feedback 6 – 1st order systems

FEEDBACK INVOLVES MEASUREMENT, DECISION MAKING BASED ON THE MEASUREMENT AND INPUT ADJUSTMENT. This changes behaviour. Here we consider the impact of proportional feedback 'K' on the closed-loop behaviour of a 1^{st} order system G(s).



$$M(s)=K, \quad G(s)=\frac{A}{Ts+1};$$

$$Y(s)=\frac{G(s)K}{1+G(s)K}R(s)=G_c(s)R(s)$$

$$G_c=\frac{KA}{Ts+KA+1}$$

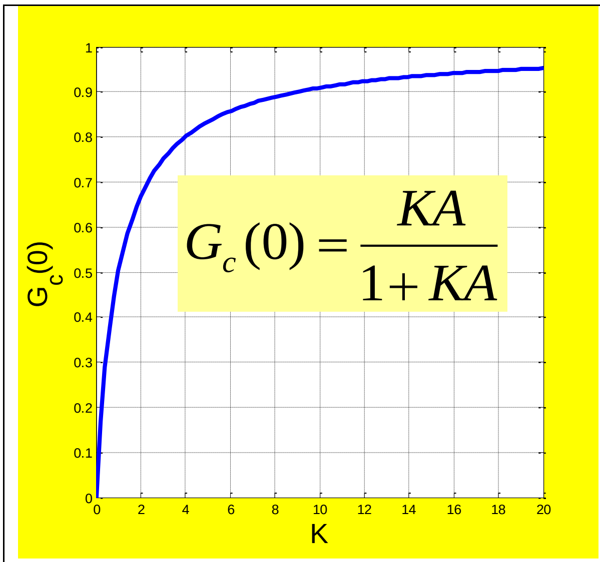
Compare the open-loop G(s) and closed-loop G_c(s) transfer functions

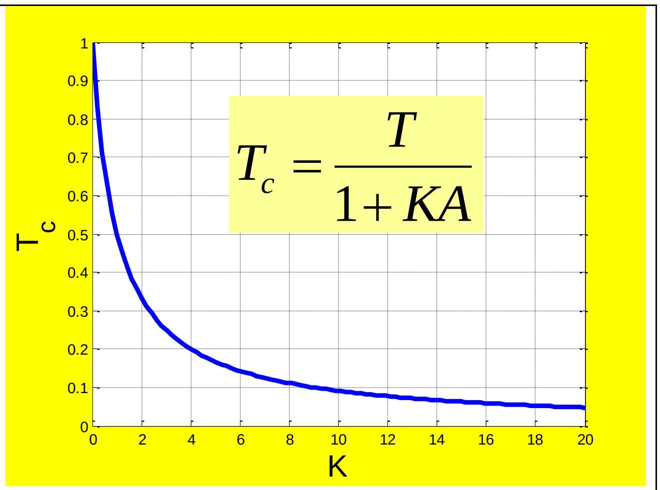
Typical measures adopted are:

1. Stability. (Is the output convergent, that is, are all poles of G_c in the LHP?)
2. Speed of response/settling time. (Taken to be about 3 times dominant time constant).
3. Closed-loop gain/offset. (Does the output reach the target and if not, how big is offset?)
4. Shape of response. (Are their oscillations and overshoots and if so, how big?)

OPEN-LOOP $G(s)=\frac{A}{Ts+1}$	**CLOSED-LOOP** $G_c(s)=\frac{KA}{Ts+KA+1}$
1. Assuming T>0, system is stable as the pole is p=-1/T is then in the LHP.	1. The closed-loop pole is given as p=-(KA+1)/T Assuming G(0)=A>0 [which is typical] and T>0 then this is guaranteed to be in LHP for any K>0, and indeed for some K<0.
2. The speed of response is directly lined to the time constant. A smaller time constant means a faster response. Here the open-loop time constant is T.	2. Closed-loop time constant is $T_c=\frac{T}{KA+1}$. Hence, using same assumptions as above, that is K>0, A>0, then the closed-loop is always faster than the open-loop and gets faster as K increases.
3. The system gain G(0) is given as 'A'.	3. The closed-loop gain $G_c(0)=\frac{KA}{KA+1}$. This is close to zero for small K and approaches one for large K.
4. As the system has a single real pole, he dynamics are a simple exponential.	4. Same comment as open-loop.

The figures below show how time constant and gain change as K increases, assuming A=1, T=1 and K>0.

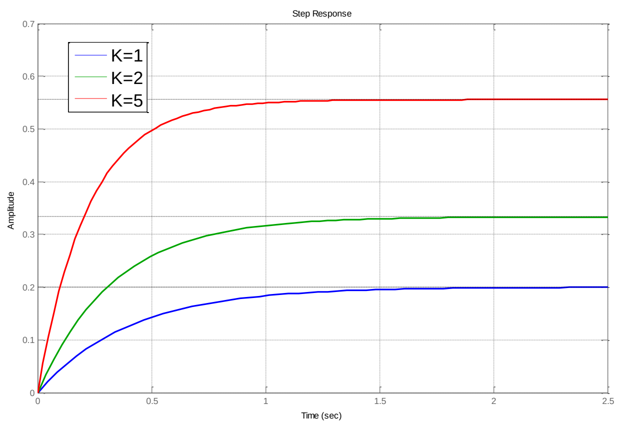




DEPENDENCE OF CLOSED-LOOP GAIN AND TIME CONSTANT EXAMPLE

$$G(s) = \frac{0.5}{s+2} = \frac{0.25}{0.5s+1} \quad \Rightarrow \quad G_c = \frac{0.25K}{0.5s+0.25K+1}; \quad T_c = \frac{0.25K}{1+0.25K}; \quad G_c(0) = \frac{0.25K}{1+0.25K}$$

K	0	1	2	5
$G_c(0)$	0	0.2	1/3	5/9
T_c	0.5	0.4	1/3	2/9



It is clear that the response speed up as K increases and aslo the steady-state gain increases.

HOWEVER, it is also clear that there is always a steady-state offset, that is the output can never get to one because the steady-state gain is less than one by definition.

Selection of the proportional gain: Points to consider when doing a design:

1. What time constant is desirable for this system?
2. What closed-loop gain is desirable? Do you need zero steady-state offset?
3. How much actuator energy/movement is available? Note that for r=1 (unity target) the initial actuator value is given by K, hence large K implies that large inputs are possible. What would you do if this is not the case?
   Is it necessary to use the constraint that K*r < max(input)?
4. How would your design be effected by parameter uncertainty? [See videos on uncertainty]

Remember, there is no single correct answer. Justify and sell your proposal!

KEY POINT: You cannot remove closed-loop offset for a 1st order system with just proportional control. A zero closed-loop offset requires that $G_c(0)=1$!