# Modelling and control summaries

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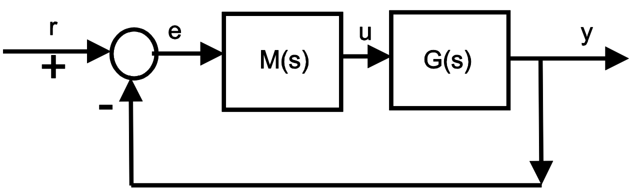
## Intro. to feedback 7 – 2nd order systems

**FEEDBACK INVOLVES MEASUREMENT, DECISION MAKING BASED ON THE MEASUREMENT AND INPUT ADJUSTMENT.** This changes behaviour. Here we consider the impact of proportional feedback 'K' on the closed-loop behaviour of a 2nd order system G(s).




$$M(s)=K,\quad G(s)=\frac{B}{s^2+As+B};$$

Open-loop

$$Y(s)=\frac{G(s)K}{1+G(s)K}R(s)=G_c(s)R(s)$$

$$G_c=\frac{KB}{s^2+As+KB+B}$$

Closed-loop

**Compare the open-loop G(s) and closed-loop $G_c(s)$ transfer functions**

1. Stability. (Is the output convergent, that is, are all poles of $G_c$ in the LHP?)
2. Speed of response/settling time. (Taken to be about 3 times dominant time constant).
3. Closed-loop gain/offset. (Does the output reach the target and if not, how big is offset?)
4. Shape of response. (Are their oscillations and overshoots and if so, how big?)

**For convenience, assume the open-loop is stable and therefore A>0 and B>0.**

**GAIN comparisons**

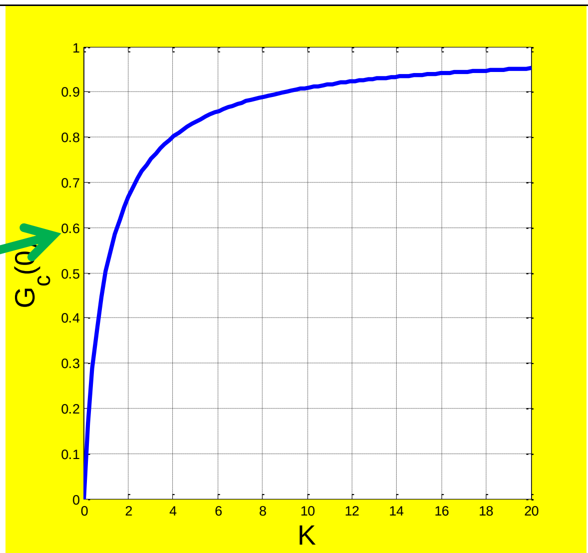
The open-loop gain has been set to one to simplify comparisons. This does not affect the key insights hereafter.

Closed-loop gain is given as

$$G_c(0)=\frac{KB}{KB+B}=\frac{K}{1+K}$$

Clearly this gain is always less than one, but increases as K increases.

**GUARANTEED a closed-loop offset!**




**STABILITY**: As the closed-loop polynomial is a quadratic, then the closed-loop is stable iff all the coefficients have the same sign.

By assumption, A>0, B>0 and K>0 and therefore the polynomial $p_c(s) = s^2+As+KB+B$ must have LHP roots.

**The closed-loop is table for all K>0!**

**SPEED OF RESPONSE AND SHAPE OF RESPONSE**: The system could have 2 distinct real roots (over-damped behaviour) or complex conjugate roots (oscillatory behaviour).

1. REAL roots with one slower than open loop behaviour if: $4B(1+K) < A^2$
   As K reduces, response gets slower.
2. Two equal roots (critical damping) if: $4B(1+K) = A^2$

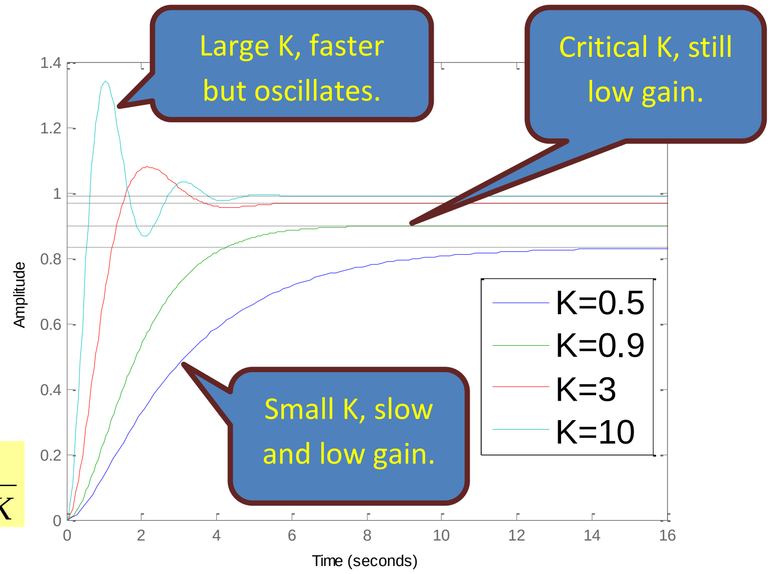
NOTE: Only possible if open-loop (K=0) is over damped so $A^2 > 4B$.

3. For larger K ( $K > (A^2 - 4B)/4B$ ), the system will have complex roots and oscillatory behaviour. The convergence rate will however be fixed as the real part of the poles will be fixed at –A/2. As K increases beyond the critical value, the oscillation gets faster/overshoot larger.

**EXAMPLE**

| K | Gain | Slowest pole | ζ |
|---|---|---|---|
| 0.5 | 0.83 | -0.37 | > 1 |
| 0.9 | 0.9 | -1 | 1 |
| 1.5 | 0.97 | -1 | < 1 |
| 3 | 0.99 | -1 | <<1 |

$$G = \frac{1}{s^2 + 2s + 0.1}; \quad G_c = \frac{K}{s^2 + 2s + 0.1 + K}$$




**Selection of the proportional gain**: Points to consider when doing a design:

1. What time constant is desirable for this system?
2. What closed-loop gain is desirable? Do you need zero steady-state offset?
3. How much overshoot and oscillation is acceptable?
4. How much actuator energy/movement is available? Note that for r=1 (unity target) the initial actuator value is given by K, hence large K implies that large inputs are possible. What would you do if this is not the case?
   Is it necessary to use the constraint that K*r < max(input)?
5. How would your design be effected by parameter uncertainty? [See videos on uncertainty]

**Remember, there is no single correct answer. Justify and sell your proposal!**

**KEY POINT:** You cannot remove closed-loop offset for a 2nd order system with just proportional control. A zero closed-loop offset requires that $G_c(0)=1$!

**ACTUATION**: For a step response, u(0)=Kr, so in practice K is limited. We cannot choose a large K to reduce offset because such a K cannot be implemented by real actuators.