Modelling and control summaries

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Offset tutorial 2

COMMENTS: Check you answers with MATLAB. Simple commands to sketch the error are something like:

```
G=tf(2,[1 1]);     M=tf([1 5],[1 3]);
Gc=feedback(1,G*M);     step(Gc);
```

QUESTION 1: A system output is determined from the following input/system pairs using Y(s)=G(s)U(s), U(s)=L[u(t)]. Find the corresponding steady-state outputs.

$$G(s)=\frac{5}{(s+1)},\quad u(t)=2$$

$$G(s)=\frac{0.2}{(s+0.1)(s^2+4s+4)},\quad u(t)=6-e^{-4t}$$

$$G(s)=\frac{-3}{(s^2+s+4)(s+0.2)},\quad u(t)=\sin 4t$$

$$G(s)=\frac{5}{(s+1)(s^2+s+4)(s+2)},\quad u(t)=e^{-2t}$$

$$G(s)=\frac{0.2}{(s-0.1)(s^2+4s+4)},\quad u(t)=2+e^{-t}$$

$$G(s)=\frac{3}{s(s^2+4s+4)(s+2)},\quad u(t)=e^{-2t}$$

$$G(s)=\frac{2}{s(s^3+s^2+s+4)},\quad u(t)=e^{-5t}$$

ANS: 10, 3, NA, 0, NA,3/16, 0.1

QUESTION 2: Find the steady-state gain of the following transfer functions.

$$G(s)=\frac{0.25}{(s+0.1)(s^2+6s+4)}$$

$$G(s)=\frac{3(s+5)}{s(s^2+s+4)(s+0.2)}$$

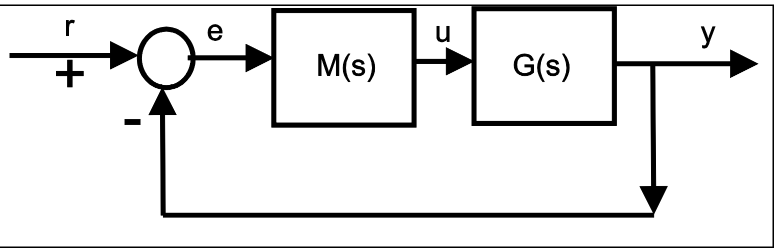
$$G(s)=\frac{2}{s(s^3-s^2+s+4)}$$

$$G(s)=\frac{2(s+4)(s+1)}{(s^2+0.1s+2)(s^2+s+4)}$$

ANS: 5/8,∞, ∞,1

ANSWERS are provided in more detail in the partner videos (offset 9-12) available on the website.

QUESTION 3: Find the steady-state offset of the following system compensator and target triples assuming a simple feedback loop. [Don't forget to confirm closed-loop stability first.]



$$G(s)=\frac{s+3}{(s+1)(s+5)}\qquad M(s)=2;\quad r(s)=\frac{3}{s}$$

$$G(s)=\frac{s-1}{(s+1)(s+2)}\qquad M(s)=0.4;\quad r(s)=\frac{1}{s}$$

$$G(s)=\frac{2}{(s-1)(s+5)}\qquad M(s)=4;\quad r(s)=\frac{1}{s}$$

$$G(s)=\frac{s+2}{(s+1)(s+4)(s+0.1)}\qquad M(s)=3;\quad r(s)=\frac{1}{s}$$

$$G(s)=\frac{s-5}{(s+1)(s+5)}\qquad M(s)=6;\quad r(s)=\frac{2}{s}$$

$$G(s)=\frac{2}{(s^3+0.8s^2+0.2s+0.1)}\qquad M(s)=3;\quad r(s)=\frac{1}{s}$$

$$G(s)=\frac{s+3}{s(s+1)(s+5)}\qquad M(s)=2;\quad r(s)=\frac{3}{s}$$

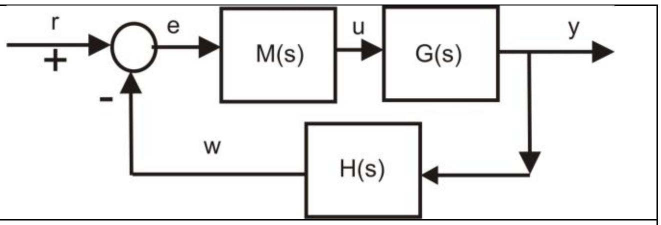
$$G(s)=\frac{s-1}{s(s+1)(s+2)}\qquad M(s)=0.4;\quad r(s)=\frac{1}{s}$$

$$G(s)=\frac{2}{s(s-1)(s+5)}\qquad M(s)=4;\quad r(s)=\frac{1}{s}$$

$$G(s)=\frac{s+2}{s(s+4)(s+0.1)}\qquad M(s)=3;\quad r(s)=\frac{1}{s}$$

ANS: 15/11, 5/4,-5/3,1/16, RHP poles so NA, RHP poles so NA, 0, RHP poles, RHP poles,0

QUESTION 4: Find the steady-state offset for r=1 of the following system, compensator, sensor and target assuming a simple feedback loop. [Don't forget to confirm closed-loop stability first.]

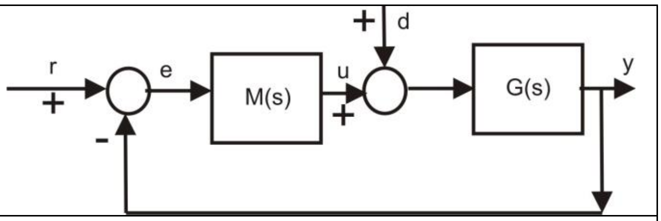


$$G(s)=\frac{s+3}{s(s+1)(s+5)};\quad H=10\frac{(s+1)}{s+10}\qquad M(s)=2;\quad r(s)=\frac{3}{s}$$

$$G(s)=\frac{s+2}{s(s+4)(s+0.1)};\quad H=\frac{s+1}{s+5}\qquad M(s)=3;\quad r(s)=\frac{1}{s}$$

ANS: 0, -4

QUESTION 5: Find the steady-state offset for unit input disturbance with the following system and compensator.



$G(s)=\frac{s-1}{(s+1)(s+2)};\quad M(s)=0.4$

$G(s)=\frac{s+3}{(s+1)(s+5)};\quad M(s)=2$

$G(s)=\frac{2}{(s-1)(s+5)};\quad M(s)=4$

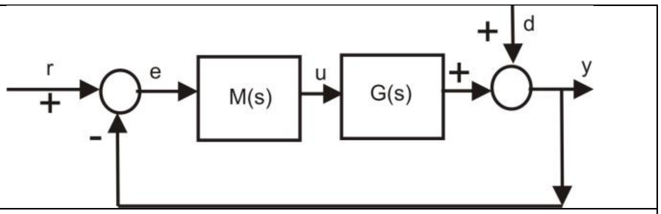
$G(s)=\frac{s+3}{s(s+1)(s+5)};\quad M(s)=2$

$G(s)=\frac{s+2}{(s+1)(s+4)(s+0.1)};\quad M(s)=3$

$G(s)=\frac{s+0.1}{s(s+1)(s+2)};\quad M(s)=0.4$

ANS: 5/8, -3/11,-2/3,-0.5,1/3.2,-2.5

QUESTION 6: Find the steady-state offset for unit output disturbance with the following system and compensator.



$G(s)=\frac{s-1}{(s+1)(s+2)};\quad M(s)=0.4$

$G(s)=\frac{s+3}{(s+1)(s+5)};\quad M(s)=2$

$G(s)=\frac{2}{(s-1)(s+5)};\quad M(s)=4$

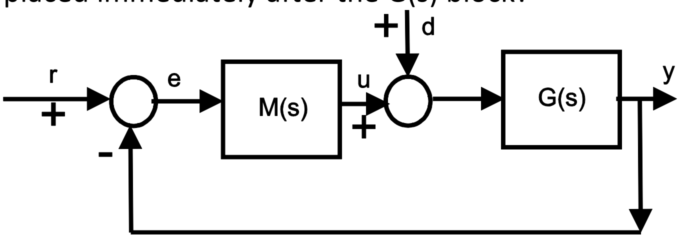
$G(s)=\frac{s+3}{s(s+1)(s+5)};\quad M(s)=2$

$G(s)=\frac{s+2}{(s+1)(s+4)(s+0.1)};\quad M(s)=3$

$G(s)=\frac{s+0.1}{s(s+1)(s+2)};\quad M(s)=0.4$

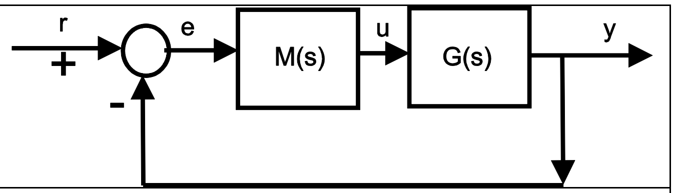
ANS: -1.25, -5/11,-5/3,0,1/19.2,0

QUESTION 7 (more bookwork in style): Derive the formulae for steady-state offset to a unit step and a unit ramp in (r) for the feedback loop give here. Use superposition to find the steady-state impact of a unit step in signal (d). How would this change if the disturbance summing junction was placed immediately after the G(s) block?



- Prove that the output (y) will only track steady-state targets if there is an integrator in either M(s) or G(s). What other condition is required?
- What requirement is there for disturbance signals to be totally rejected, that is to have no steady-state impact on the output?
- Comment on how the requirements might change for nested loops.

QUESTION 8: Find the steady-state offset of the following system/compensator assuming a simple feedback loop and a ramp target. [Don't forget to confirm closed-loop stability first.]



$$G(s)=\frac{s+3}{(s+1)(s+5)} \quad M(s)=2;\quad r(s)=\frac{3}{s^2}$$

$$G(s)=\frac{s-1}{(s+1)(s+2)} \quad M(s)=0.4;\quad r(s)=\frac{1}{s^2}$$

$$G(s)=\frac{2}{(s-1)(s+5)} \quad M(s)=4;\quad r(s)=\frac{1}{s^2}$$

$$G(s)=\frac{s+2}{(s+1)(s+4)(s+0.1)} \quad M(s)=3;\quad r(s)=\frac{1}{s^2}$$

$$G(s)=\frac{s+3}{s(s+1)(s+5)} \quad M(s)=2;\quad r(s)=\frac{3}{s^2}$$

$$G(s)=\frac{s+0.1}{s(s+1)(s+2)} \quad M(s)=0.4;\quad r(s)=\frac{1}{s^2}$$

$$G(s)=\frac{2(s+3)}{s(s-1)(s+5)} \quad M(s)=12;\quad r(s)=\frac{1}{s^2}$$

$$G(s)=\frac{s+2}{s(s+4)(s+0.1)} \quad M(s)=3;\quad r(s)=\frac{1}{s}$$

$$G(s)=\frac{s+3}{s(s+1)(s+5)};\quad M=20\frac{s+2}{s}$$

ANS: ∞,∞,∞,∞, 15/6,50,-5/72,0.4/6,0

QUESTION 9: Find the steady-state offset between a unit target and the output for the following system compensator pairs. Check your answers with MATLAB.

$$\left\{\begin{matrix}G=\frac{1}{s^2+2s+1}\\ K=2\end{matrix}\right\} \quad \left\{\begin{matrix}G=\frac{1}{s^3+3s^2+3s+1}\\ K=0.5\end{matrix}\right\} \quad \left\{\begin{matrix}G=\frac{1}{(s+10)(s+2)}\\ K=8\end{matrix}\right\} \quad \left\{\begin{matrix}G=\frac{1}{s^2+2s+1}\\ K=\frac{0.4}{s}\end{matrix}\right\}$$

$$\left\{\begin{matrix}G=\frac{1}{s^2+2s+3}\\ K=4\end{matrix}\right\};\left\{\begin{matrix}G=\frac{1}{s^3+5s^2+3s+2}\\ K=0.6\end{matrix}\right\};\left\{\begin{matrix}G=\frac{(s+3)}{(s+1)(s+2)(s+4)}\\ K=8\end{matrix}\right\} \quad \left\{\begin{matrix}G=\frac{1}{s^3+3s^2+3s+1}\\ K=\frac{0.2}{s}\end{matrix}\right\}$$

$$\left\{\begin{matrix}G=\frac{s+3}{s^2+2s+1}\\ K=\frac{4}{s}\end{matrix}\right\};\left\{\begin{matrix}G=\frac{1}{s^3+5s^2+3s+2}\\ K=0.6\frac{(s+2)}{s}\end{matrix}\right\};\left\{\begin{matrix}G=\frac{(s+3)}{s^3+5s^2+3s}\\ K=0.8\end{matrix}\right\}$$

- How would these results change if the target was a unit ramp?
- For all the examples above, use MATLAB to investigate how offset and performance varies as you change the scalar gain K. Give some generic conclusions based upon what you observe.