

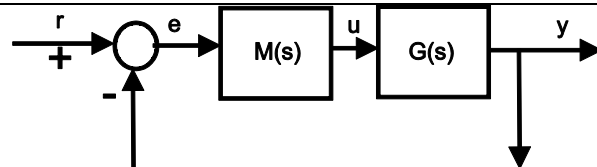
Modelling and control summaries



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Offset and gain tutorial 1

This summary assumes a standard feedback loop and unless specifically stated otherwise, the target $r=1$. **DON'T forget to check stability first!**



1. A system output is determined from the following input/system pairs using $Y(s)=G(s)U(s)$, $U(s)=L[u(t)]$. Find the corresponding steady-state output

$$\left\{ \begin{array}{l} G(s) = \frac{5}{(s+1)} \\ u(t) = 2 \end{array} \right\} \quad \left\{ \begin{array}{l} G(s) = \frac{0.2}{(s+0.1)(s^2+4s+4)} \\ u(t) = 6 - e^{-4t} \end{array} \right\} \quad \left\{ \begin{array}{l} G(s) = \frac{-3}{(s^2+s+4)(s+0.2)} \\ u(t) = \sin 4t \end{array} \right\}$$

$$\left\{ \begin{array}{l} G(s) = \frac{5}{(s+1)(s^2+s+4)(s+2)} \\ u(t) = e^{-2t} \end{array} \right\} \quad \left\{ \begin{array}{l} G(s) = \frac{0.2}{(s-0.1)(s^2+4s+4)} \\ u(t) = 2 + e^{-t} \end{array} \right\}$$

$$\left\{ \begin{array}{l} G(s) = \frac{3}{s(s^2+4s+4)(s+2)} \\ u(t) = e^{-2t} \end{array} \right\} \quad \left\{ \begin{array}{l} G(s) = \frac{2}{s(s^3+s^2+s+4)} \\ u(t) = e^{-5t} \end{array} \right\}$$

ANSWERS in order: 10,3,NA,0,NA,3/16,NA

HINT: You can form $Y(s)=G(s)U(s)$ and apply the FVT directly (if signal is convergent).

2. Find the system steady-state gains

$$G(s) = \frac{2}{s+1}; \quad G(s) = \frac{s+5}{(s+3)};$$

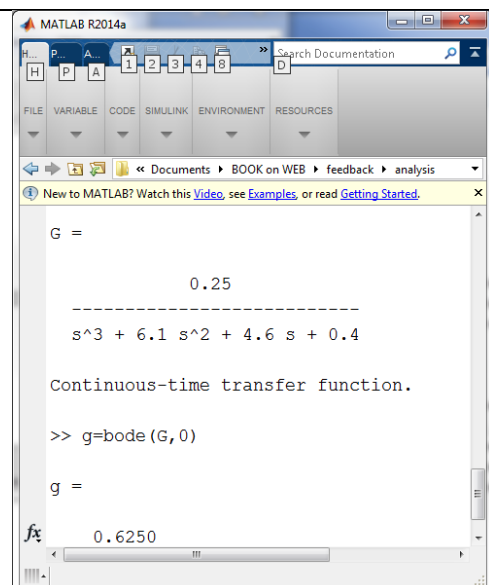
$$G(s) = \frac{s+5}{s(s+3)}; \quad G(s) = \frac{s+2}{s(s+3)(s+1)};$$

$$G(s) = \frac{0.25}{(s+0.1)(s^2+6s+4)}; \quad G(s) = \frac{3(s+5)}{s(s^2+s+4)(s+0.2)};$$

$$G(s) = \frac{2}{s(s^3-s^2+s+4)}; \quad G(s) = \frac{2(s+4)(s+1)}{(s^2+0.1s+2)(s^2+s+4)}$$

Answers in order: 2,5/3,∞,∞,0.625,∞,∞,1

You can check your answers using bode.m in MATLAB as shown here.



3. Find the steady-state offset for the following system/compensator pairs

$G(s) = \frac{2}{(s+1)(s+5)}$; $M(s) = \frac{s+5}{s+3}$	$G(s) = \frac{6}{s(s+2)(s+4)}$; $M(s) = 1$
$G(s) = \frac{2}{(s+4)(s+1)}$; $M(s) = 6\frac{s+3}{s+10}$	$G(s) = \frac{6}{(s+2)(s+4)}$; $M(s) = \frac{20}{s}$
$G(s) = \frac{2}{s(s+4)(s+1)}$; $M(s) = 6\frac{s+3}{s+10}$	$G(s) = \frac{3s+1}{(s-2)(s+5)}$; $M(s) = \frac{100}{s}$
$G(s) = \frac{6}{(s+2)(s+4)}$; $M(s) = 1$ or 20 or 100	$G(s) = \frac{s-1}{(s+1)(s+2)}$; $M(s) = 0.4$
$G(s) = \frac{s+3}{(s+1)(s+5)}$; $M(s) = 2$	$G(s) = \frac{2}{(s-1)(s+5)}$; $M(s) = 2$ or 4
$G(s) = \frac{s+2}{(s+1)(s+4)(s+0.1)}$; $M(s) = 3$	$G(s) = \frac{s-5}{(s+1)(s+5)}$; $M(s) = 6$
$G(s) = \frac{2}{s^3 + 0.8s^2 + 0.2s + 0.1}$; $M(s) = 3$	$G(s) = \frac{s-1}{s(s+1)(s+5)}$; $M(s) = 0.4$
$G(s) = \frac{s+2}{s(s+4)(s+0.1)}$; $M(s) = 3$	Are there any clear patterns in your answers?

Checking your answers: Students must become competent in using tools like MATLAB to reinforce their learning and check numerical answers. Some typical commands are given here.

```

1 - clf reset
2 - G=tf(2,poly([-1 -5]));
3 - M=tf([1 5],[1 3]);
4 - Gce=feedback(1,G*M);
5 - p=pzmap(Gce);
6 - max_real_part_of_poles=max(real(p))
7 - offset=bode(Gce,0)
    
```

max_real_part_of_poles =
-2.0000

offset =
0.6000

Check all poles in LHP otherwise divergent output

Valid if above answer negative

Use **>>step(Gce)** to plot the response.

This will confirm it is convergent and also allow you to see asymptotic value.

