This summary assumes that any implied signals are convergent and hence the final value theorem (FVT) can be applied – see offset 1.

**DEFINITION OF SYSTEM STEADY-STATE GAIN**

Assume that the system input $u(t)$ is constant. The steady-state gain is the ratio of the asymptotic output signal to the input signal assuming the output is convergent. For convenience use $u(t)=1$, so gain becomes asymptotic output.

$$\lim_{s \to 0} \frac{G(s)}{s} = \lim_{s \to 0} G(s) = G(0)$$

**REMARK:** System gain $G(0)$ is well defined even for unstable $G(s)$ because a convergent $u(t)$ can be chosen to cancel any divergent dynamics and hence give a convergent output.

What is the system gain for the following transfer functions?

$$H(s) = \frac{A}{s+1} \quad \Rightarrow \quad H(0) = A$$

$$G(s) = \frac{5}{(s+1)(s^2 + s + 4)(s + 2)} \quad \Rightarrow \quad G(0) = \frac{5}{8}$$

$$M(s) = \frac{4}{s(s+3)} \quad \Rightarrow \quad M(0) = \infty$$

You can verify these for $H$, $G$, $M$ by using step.m in MATLAB and finding asymptotic output.

**DEFINITION OF SYSTEM CLOSED-LOOP STEADY-STATE GAIN**

First determine the closed-loop transfer functions and decide which gain you want, that is the gain between which signals.

1. Closed-loop gain is typically defined by linking the target $r$ to the output $y$, that is $G_c$.
2. Offset is $e=(r-y)$ and more often users are interested in this as it is the steady-state error between target and output, that is $G_{ce}$.

**REMARK:** This note will consider the computation of both system gain and offset. For convenience, offset is normally defined for a unit target, that is $r=1$. HENCE, as gain is equivalent to asymptotic output signal value, then:

$$GAIN=G_c(0)$$

$$OFFSET=G_{ce}(0)$$
SHORTCUT: \[ G_c(0) = \left( \frac{G(0)M(0)}{1+G(0)M(0)} \right); \quad G_{ce}(0) = \left( \frac{1}{1+G(0)M(0)} \right) \]

EXAMPLES - assuming a standard closed-loop configuration, find the steady-state system gain and steady-state offsets (for unit target \( r=1 \)) for the following.

FIRST you must check that the closed-loop is stable (by all means confirm with MATLAB or otherwise).

\[
G(s) = \frac{3}{(s-1)(s+5)}; \quad M(s) = 2
\]

\[
G_c(0) = \left( \frac{6}{1+6/5} \right) = 6; \quad G_{ce}(0) = \left( \frac{1}{1+6/5} \right) = -5
\]

\[
G(s) = \frac{s+6}{(s+4)(s+2)}; \quad M(s) = 2
\]

\[
G_c(0) = \left( \frac{12/8}{1+12/8} \right) = \frac{3}{5}; \quad G_{ce}(0) = \left( \frac{1}{1+12/8} \right) = \frac{2}{5}
\]

\[
\left\{ G(s) = \frac{1}{s-4}; \quad M(s) = 2 \right\} \Rightarrow G_c = \frac{2}{s-4+2}
\]

What M would give a LHP pole – try this and then find gain/offset?

\[
G(s) = \frac{s+1}{s(s+3)}; \quad M(s) = 4
\]

\[
G_c(0) = \left( \frac{4/0}{1+4/0} \right) = 1; \quad G_{ce}(0) = \left( \frac{1}{1+4/0} \right) = 0
\]

Note use of integrator has made the steady-state gain \( G(0) \to \infty \) and this affects result.

SUMMARY:
1. System steady-state gain can be computed by substituting \( s=0 \) into a transfer function.
2. The steady-state gain of a loop can be computed by substituting \( s=0 \) into a closed-loop transfer function.
3. Steady-state output is gain \( \times \) steady state input.
4. Signals must be convergent so check for stability.
5. If a transfer function contains an integrator, the steady-state gain is infinite.

CHALLENGE: How would system closed-loop gain and offset calculations change for a feedback loop with sensor dynamics such as the one below?