Modelling and control summaries

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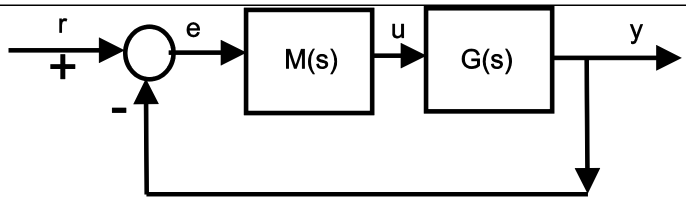
Offset 3 – steady-state offset

DEFINITION OF CLOSED-LOOP SYSTEM STEADY-STATE GAIN and OFFSETS

Assume that the loop input r(t) is constant. For convenience use r(t)=1 so offset always defined relative to a unit target.

$$y=\underbrace{\left(\frac{GM}{1+GM}\right)}_{G_c}r; \quad e=\underbrace{\left(\frac{1}{1+GM}\right)}_{G_{ce}}r$$

Using the FVT



$$Gain=\frac{G(0)M(0)}{1+G(0)M(0)}; \quad offset=\frac{1}{1+G(0)M(0)}$$

REMINDER: FVT and hence formulae only apply when the loop is stable!

EXAMPLE 1: Find the steady-state offset for the following system

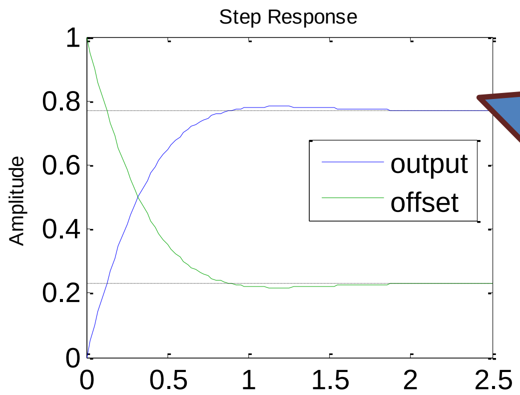
$$G=\frac{2}{s+1}; \quad M(s)=\frac{s+5}{s+3}$$

$$G(0)=2; \quad M(0)=5/3$$

$$Gain=\frac{10/3}{1+10/3}=\frac{10}{13}; \quad offset=\frac{1}{1+10/3}=\frac{3}{13}$$

This result can be confirmed with MATLAB, e.g.:

```
G=tf(2,[1 1]);
M=tf([1 5],[1 3]);
Gc=feedback(G*M,1);
Gce=feedback(1,G*M);
step(Gc,Gce,2.5);
```



Clearly the output is 10/13 and not close to r=1.

Offset =3/13 is not zero.

OBSERVATION: There is a closed-loop offset. In practice this feedback law is not good!

ANALYSIS OF SYSTEM CLOSED-LOOP OFFSET

1. If M(0) and G(0) are both finite then the steady-state offset cannot be zero.
2. This is obvious from the formulae.

$$offset=\left(\frac{1}{1+G(0)M(0)}\right)$$

EXAMPLE 2: Find the steady-state offset for the following system

$$G=\frac{2}{(s+1)(s+4)}; \quad M(s)=6\frac{s+3}{s+10}$$

$$G(0)=0.5; \quad M(0)=1.8$$

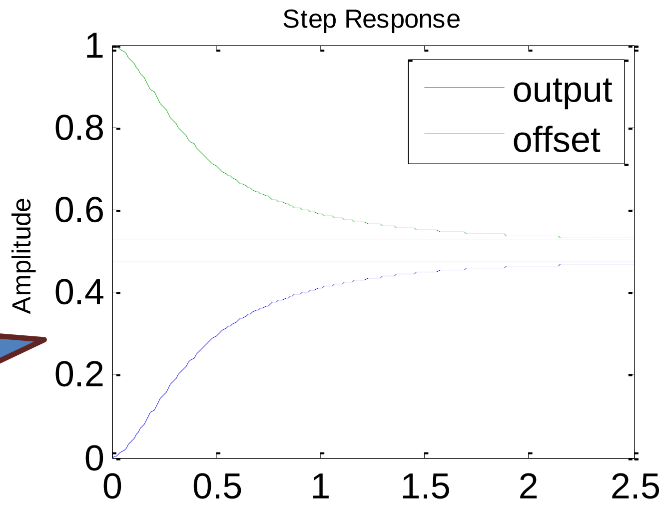
$$Gain=\frac{0.9}{1+0.9}=0.47; \quad offset=\frac{1}{1+0.9}=0.53$$

This result can be confirmed with MATLAB, e.g.:

```
G=tf(2,[1 5 4]);
M=6*tf([1 3],[1 10]);
Gc=feedback(G*M,1);
Gce=feedback(1,G*M);
step(Gc,Gce,2.5);
```

Clearly the output is 0.47 and not close to r=1.

Offset =0.53 is not zero.



OBSERVATION: There is a closed-loop offset. In practice this feedback law is not good!

SUMMARY:
For systems G(s) and compensators M(s) in a simple feedback loop and with finite steady-state gain, the closed-loop will have a steady-state offset which in many cases could be quite large.