Modelling and control summaries

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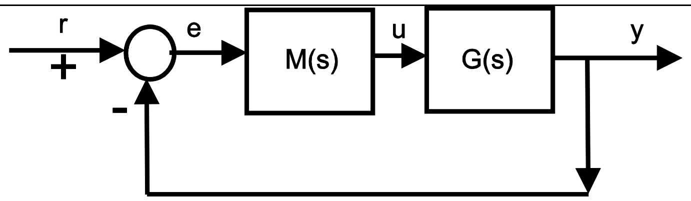
Offset 4 – steady-state offset with integrator

DEFINITION OF CLOSED-LOOP SYSTEM OFFSET

Assume that the loop input r(t) is constant. For convenience use r(t)=1 so offset always defined relative to a unit target.

$$y=\underbrace{\left(\frac{GM}{1+GM}\right)}_{G_c}r;\quad e=\underbrace{\left(\frac{1}{1+GM}\right)}_{G_{ce}}r$$



Using the FVT $offset=r-y=\lim_{s\to 0}\frac{1}{1+G(s)M(s)}$

REMINDER: FVT and hence formulae only apply when the loop is stable!

OFFSET is zero if and only if either the system or compensator have infinite gain in steady-state, that is:

$$\lim_{s\to 0}\frac{1}{1+G(s)M(s)}=0 \quad\Rightarrow\quad \lim_{s\to 0}G(s)M(s)=\infty$$

Either $\lim_{s\to 0}G(s)=\infty$ or $\lim_{s\to 0}M(s)=\infty$ (or indeed both).

Remember however the loop must be stable.

WHICH SYSTEMS HAVE INFINITE STEADY-STATE GAIN?

Find the steady-state gain of the following transfer functions

$$G(s)=\frac{2}{s+1}\quad\Rightarrow\quad \lim_{s\to 0}G(s)=2 \qquad G(s)=\frac{s+2}{s(s+1)(s+3)}\quad\Rightarrow\quad \lim_{s\to 0}G(s)=\frac{2}{0}=\infty$$

$$G(s)=\frac{s+5}{s+3}\quad\Rightarrow\quad \lim_{s\to 0}G(s)=5/3 \qquad G(s)=\frac{s+5}{s(s+3)}\quad\Rightarrow\quad \lim_{s\to 0}G(s)=\frac{5}{0}=\infty$$

Systems have infinite steady-state gain if and only if they include an integrator, that is a pole at the origin.

Find the corresponding closed-loop transfer functions for the systems listed above

$$G(s)=\frac{2}{s+1}\quad\Rightarrow\quad G_c=\frac{G}{1+G}=\frac{2}{s+3};\quad G_c(0)\neq 1 \qquad G(s)=\frac{s+2}{s(s+1)(s+3)}\quad\Rightarrow\quad G_c=\frac{G}{1+G}=\frac{s+2}{s^3+4s^2+4s+2};\quad G_c(0)=1$$

$$G(s)=\frac{s+5}{s+3}\quad\Rightarrow\quad G_c=\frac{G}{1+G}=\frac{s+5}{2s+8};\quad G_c(0)\neq 1 \qquad G(s)=\frac{s+5}{s(s+3)}\quad\Rightarrow\quad G_c=\frac{G}{1+G}=\frac{s+5}{s^2+3s+5};\quad G_c(0)=1$$

It is clear that **where an integrator has been included**, the closed-loop steady-state gain from targets to outputs is 1.

SUMMARY: There is no closed-loop offset to targets when an integrator is included in the loop, assuming of course that the closed-loop is stable.

EXAMPLE 1: Find the steady-state offset for the following system (remember you also need to verify that all the poles are in the LHP).

$$G=\frac{2}{s(s+1)(s+4)};\quad M(s)=6\frac{s+3}{s+10}$$

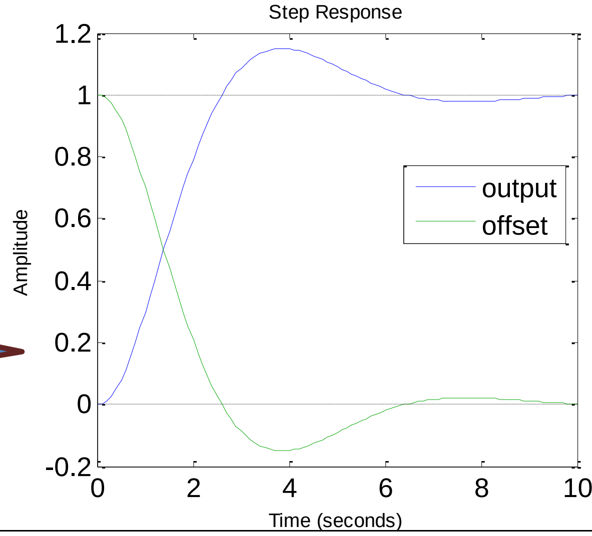
$$G(0)=\infty \quad \Rightarrow \quad Gain=1; \quad offset=0$$

This result can be confirmed with MATLAB, e.g.:

```
G=tf(2,poly([0 -1 -4]));
M=6*tf([1 3],[1 10]);
Gc=feedback(G*M,1);
Gce=feedback(1,G*M);
step(Gc,Gce,10);
```

Clearly the output is 1.

Offset =0.



OBSERVATION: There is a closed-loop offset. In practice this feedback law is not good!

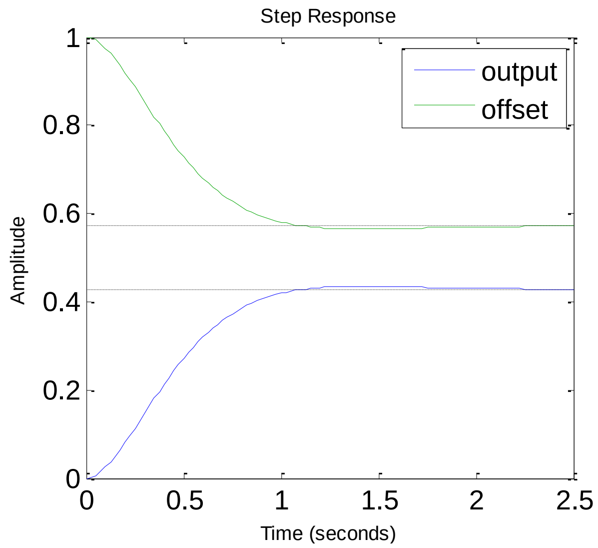
EXAMPLE 2: Compare the closed-loop behaviour of a system with and without an integrator.

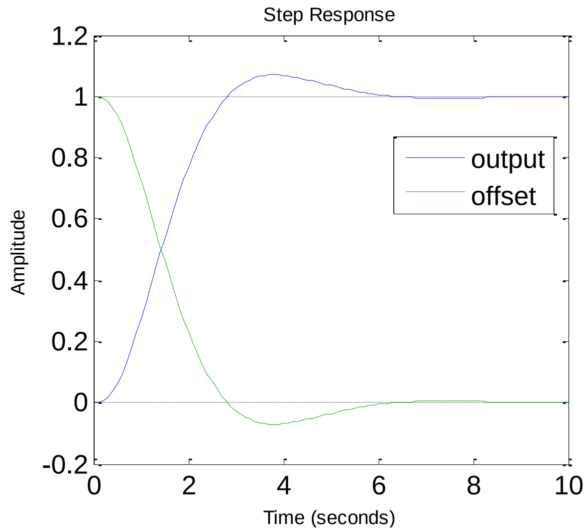
$$G=\frac{6}{(s+2)(s+4)};\quad M(s)=1$$

NO INTEGRATOR – LARGE OFFSET

$$G=\frac{6}{s(s+2)(s+4)};\quad M(s)=1$$

WITH INTEGRATOR, NO OFFSET BUT MUCH SLOWER CONVERGENCE





EXAMPLE 3: Find the offset for the following

$$G=\frac{3s+1}{(s-2)(s+5)};\quad M(s)=\frac{1}{s}$$

Even though it includes an integrator, the closed-loop is unstable, so there is no output convergent. Offset ➔ ∞