Modelling and control summaries

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Offset 5 – offset with input disturbance

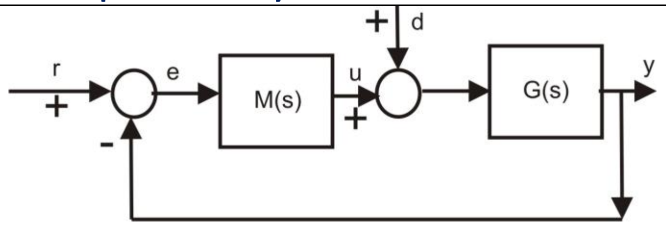
CLOSED-LOOP SYSTEM STEADY-STATE OFFSET TO INPUT DISTURBANCES

The aim is to totally reject disturbances so the dependence of y on d should → 0

Assume that the loop input d(t) is constant and ignore r(t).

$$y=\underbrace{\left(\frac{G}{1+GM}\right)}_{G_c}d;\quad e=\underbrace{\left(\frac{-G}{1+GM}\right)}_{G_{ce}}d$$

$$offset=\lim_{s\to 0}\frac{-G(s)}{1+G(s)M(s)}$$



OFFSET WITH NO INTEGRATOR

$$offset=\frac{-G(0)}{1+G(0)M(0)}$$

The offset is finite if G(0) and M(0) is finite and therefore the disturbance is not rejected.

OFFSET WITH INTEGRATOR in G(s)

$$\lim_{s\to 0}G(s)=\infty,\quad \lim_{s\to 0}M(s)=M(0),$$

$$offset=\lim_{s\to 0}\frac{-G(s)}{1+G(s)M(s)}=\lim_{s\to 0}\frac{-G(s)}{G(s)M(0)}=\frac{-1}{M(0)}$$

If the integrator is in G(s), then an input disturbance is not rejected at the output and the resulting offset is linked solely to the steady-state gain of the compensator.

OFFSET WITH INTEGRATOR in M(s)

$$\lim_{s\to 0}G(s)=G(0)\quad \lim_{s\to 0}M(s)=\infty,$$

$$offset=\lim_{s\to 0}\frac{-G(s)}{1+G(s)M(s)}=\lim_{s\to 0}\frac{-G(0)}{G(0)M(s)}=0$$

If the integrator is in M(s), then an input disturbance is rejected at the output.

SUMMARY: In order to reject input disturbances, the integrator must be in the compensator.

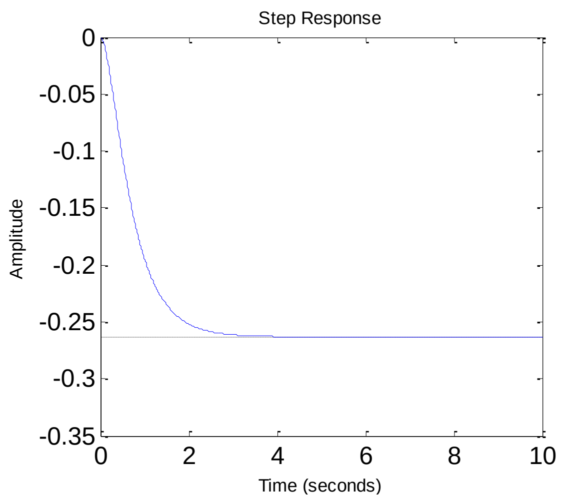
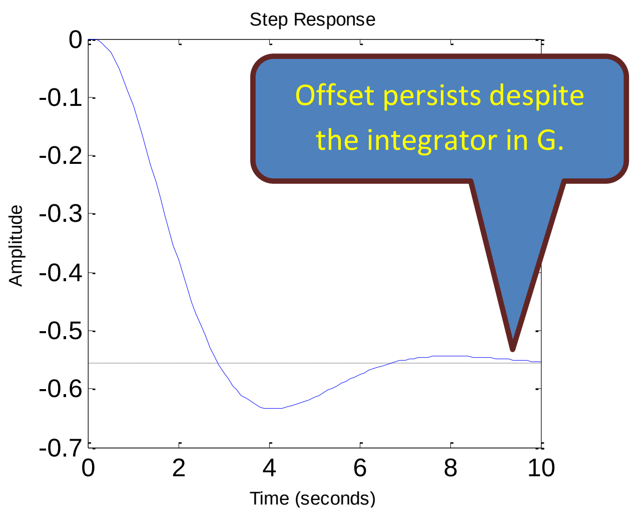
EXAMPLE 1: Find the steady-state offset for the following systems (remember you also need to verify that all the poles are in the LHP) when subject to a unit input disturbance

$$G=\frac{2}{(s+1)(s+4)};\quad M(s)=6\frac{s+3}{s+10}$$

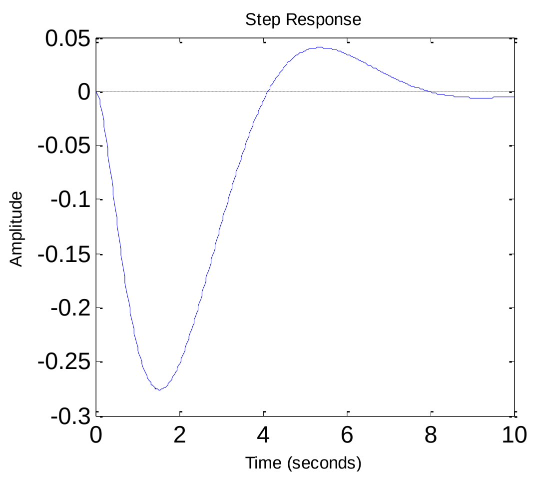
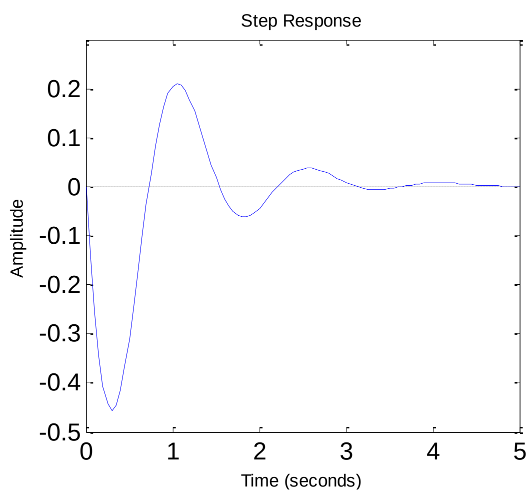
$$offset=\frac{-G(0)}{1+G(0)M(0)}=\frac{-0.5}{1+0.9}\approx -0.26$$

$$G=\frac{2}{s(s+1)(s+4)};\quad M(s)=6\frac{s+3}{s+10}$$

$$offset=\frac{-1}{M(0)}=\frac{-1}{1.8}\approx -0.55$$

EXAMPLE 2: The first one is similar to example 1 except that now the integrator is moved to the compensator instead of the system.

$G=\frac{2}{(s+1)(s+4)};\quad M(s)=6\frac{s+3}{s(s+10)}$ $offset=\lim_{s\to 0}\frac{-G(s)}{1+G(s)M(s)}=\lim_{s\to 0}\frac{-G(0)}{G(0)M(s)}=0$	$G=\frac{3s+1}{(s-2)(s+5)};\quad M(s)=\frac{10}{s}$ [Note verified that closed-loop is stable]
	

CONCLUSIONS

Input disturbances can only be rejected in steady-state if there is an integrator in the compensator.
Obviously this assumes, as ever, that the closed-loop system is stable.