Modelling and control summaries

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Offset 7 – impact of a sensor on offset

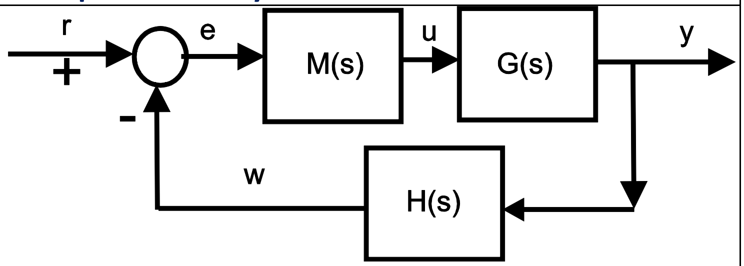
CLOSED-LOOP SYSTEM STEADY-STATE OFFSET TO OUTPUT DISTURBANCES

The aim is to totally reject disturbances so the dependence of y on d should → 0

Ultimately we are interested in the signal y and the error between this and the target r (here use r =1 for convenience).

$$y = \underbrace{\left(\frac{GM}{1+GMH}\right)}_{G_c} r;$$

$$offset = 1 - \lim_{s\to 0} \frac{G(s)M(s)}{1+G(s)M(s)H(s)}$$



This note will assume that an integrator is included in M(s) or G(s) as this would be normal practice.

There is never an integrator in sensor H(s)!

ASSUMING THAT $\lim_{s\to 0} G(s)M(s) = \infty$, then $\lim_{s\to 0} \frac{G(s)M(s)}{1+G(s)M(s)H(s)} \equiv \lim_{s\to 0} \frac{1}{H(s)}$

HENCE $$offset = 1 - \lim_{s\to 0} \frac{1}{H(s)} = \frac{H(0)-1}{H(0)}$$ → **offset=0, iff H(0)=1.**

SUMMARY: A zero offset requires $\lim_{s\to 0} \frac{G(s)M(s)}{1+G(s)M(s)H(s)} = 1$

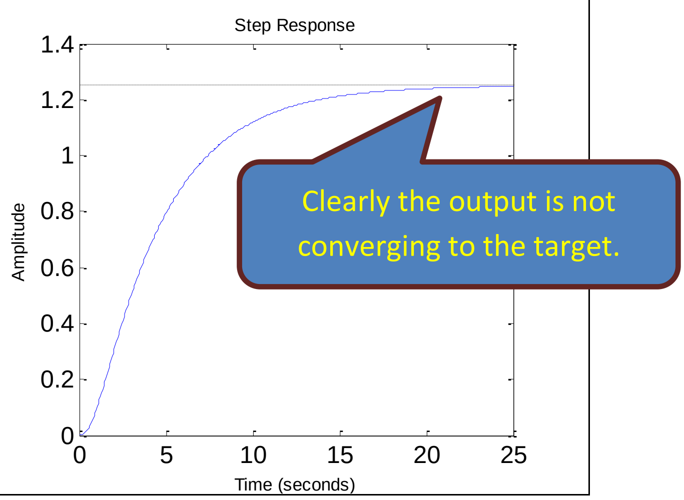
With an integrator in G(s) or M(s), this is equivalent to the requirement that H(0)=1.

EXAMPLE 1: Find the steady-state offset for the following system (remember you also need to verify that all the poles are in the LHP) for a unit target.

$$G = \frac{2}{(s+2)(s+4)}; \quad M(s) = \frac{1}{s};$$

$$H(s) = \frac{8}{s+10}$$

$$offset = \frac{H(0)-1}{H(0)} = \frac{-0.2}{0.8} = -0.25$$

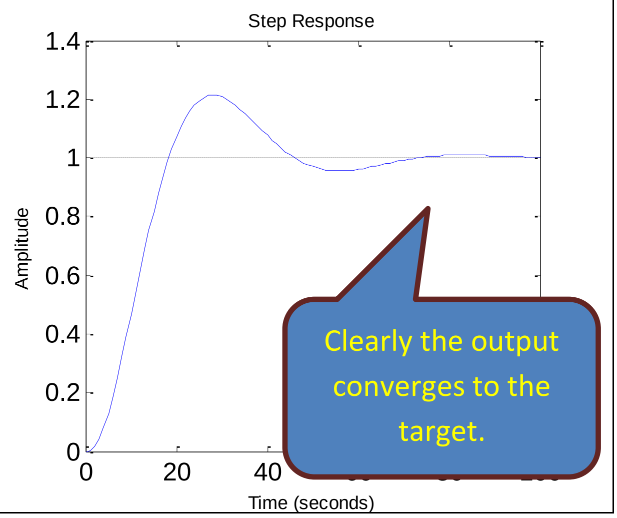


EXAMPLE 2: Find the steady-state offset for the following system (remember you also need to verify that all the poles are in the LHP) for a unit target.

$$G = \frac{0.2}{(s+1)(s+0.2)}; \quad M(s) = \frac{0.1}{s};$$

$$H(s) = \frac{0.4}{s+0.4}$$

$$\boxed{offset = \frac{H(0)-1}{H(0)} = 0}$$



DISCUSSION: For practical reasons a sensor may not have a steady-state gain of unity. This could be because of a change of variable, for example measuring a temperature in degrees and representing this as a voltage.

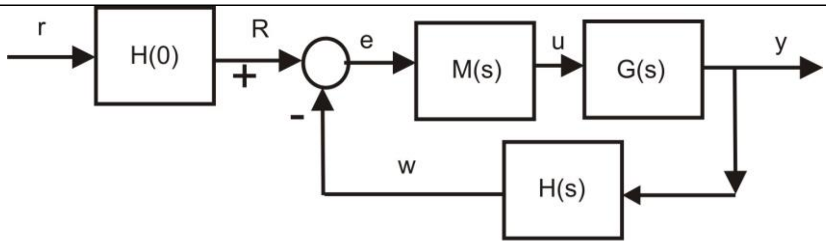
In such a case the solution is relatively obvious, ensure that the target is represented in the equivalent units. In terms of a block diagram, this could be represented by a feedforward block with a known gain to match that of the sensor.

Assuming there is an integrator in M(s) or G(s) then e(t)➔0.

This means that

$$\lim_{t\to\infty}[R(t)-w(t)]=0$$

However



$$\left.\begin{aligned}\lim_{t\to\infty} R(t) &= H(0)r \\ \lim_{t\to\infty} w(t)] &= H(0)[\lim_{t\to\infty} y(t)]\end{aligned}\right\} \Rightarrow \lim_{t\to\infty}[R(t)-w(t)] = H(0)[r-\lim_{t\to\infty} y(t)] = 0$$

HENCE: $[r-\lim_{t\to\infty} y(t)] = 0$, that is, there is no offset.

EXAMPLE 3: Find the steady-state offset for the following system for a unit target.

$$G = \frac{2}{(s+2)(s+4)}; \quad M(s) = \frac{1}{s};$$

$$H(s) = \frac{16}{s+20}$$