Modelling and control summaries

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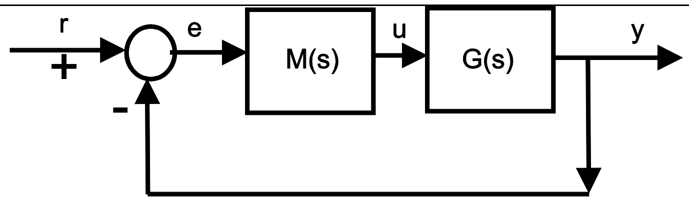
Offset 8 – tracking ramp targets

COMMENTS: A ramp signal such as r=t diverges to infinity and hence one cannot talk about output convergence to a finite value, instead the focus has to be on the convergence of the error term r-y. Ramps could represent circular motion or only operate for short periods.
For convenience this note uses a unit ramp so that $L[r(t)] = 1/s^2$.

DEFINITION OF CLOSED-LOOP SYSTEM OFFSET TO A UNIT RAMP TARGET

Using the results of earlier notes but using $r(s)=1/s^2$ the error is given as:

$$e_{ramp} = \frac{1}{1+GM}\frac{1}{s^2}$$



Hence: $offset = \lim_{s\to 0} \frac{1}{1+G(s)M(s)}\frac{1}{s} = \lim_{s\to 0} \frac{1}{sG(s)M(s)}$ **[Iff signal is convergent!]**

The offset can be characterised by 3 different possibilities

1. $\lim_{s\to 0} \frac{1}{sG(s)M(s)} = \infty$ This will occur if both G(0) and M(0) are finite.
2. $\lim_{s\to 0} \frac{1}{sG(s)M(s)} \neq 0 \; but \;\; finite$ This will occur if either G(s) or M(s) include an integrator.
3. $\lim_{s\to 0} \frac{1}{sG(s)M(s)} = 0$ This will occur if there are two integrators shared across G(s) and M(s).

REMINDER: The loop must be stable for any of this analysis to be valid.

SUMMARY:

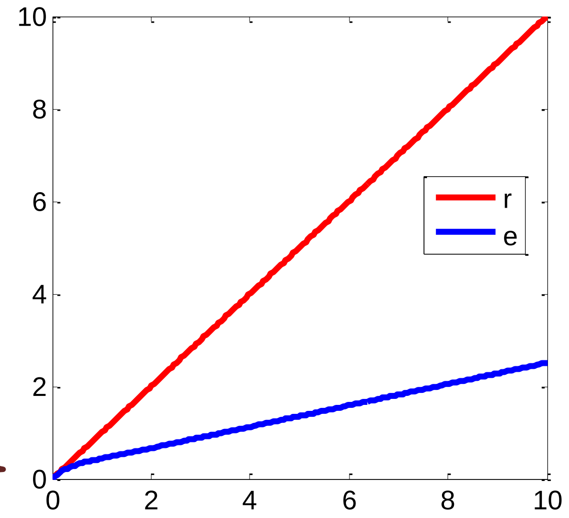
1. For simple loop structures, system steady-state offset to a unit ramp can be eliminated if between them the controller and system include at least two integrators.
2. With a single integrator the steady-state error is finite.
3. With no integrator, the offset grows without bound.

WARNING: Including two integrators will lead to poorly conditionned loops in general.

EXAMPLE 1 – no integrator: Find the steady-state offset for the following system to a unit ramp. $G = \frac{2}{(s+1)}; \quad M(s) = \frac{s+5}{s+3};$

$$\lim_{s\to 0} \frac{1}{sG(s)M(s)} = \frac{1}{0\times 2\times (5/3)} = \infty$$

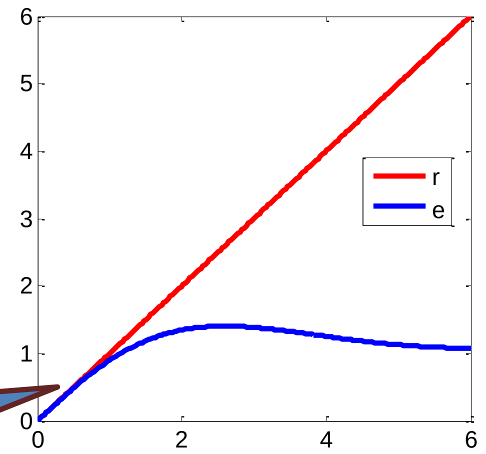
Clearly the error is diverging



EXAMPLE 2 – one integrator: Find the steady-state offset for the following system to a unit ramp. $G = \frac{2}{s(s+1)(s+4)}; \quad M(s) = 6\frac{s+3}{s+10};$

$$\lim_{s\to 0} \frac{1}{sG(s)M(s)} = \frac{1}{s\times (2/4s)\times (1.8)} = \frac{1}{0.9}$$

Clearly the error is convergent, but not to zero.



EXAMPLE 3 – two integrators: Find the steady-state offset for the following system to a unit ramp. $G = \frac{6(s+1)}{s(s+4)}; \quad M(s) = \frac{s+1}{s};$

$$\lim_{s\to 0} \frac{1}{sG(s)M(s)} = 0$$

Clearly the error is convergent to 0.