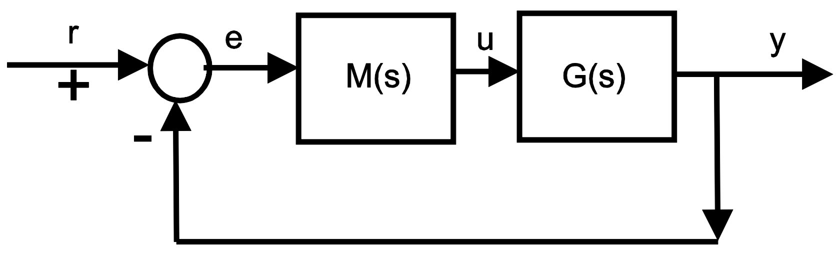
Modelling and control summaries

by Anthony Rossiter

Simple feedback 2: Proportional design

This brief summary gives a heuristic approach to proportional design, with an overall PI compensator design. The idea is to identify, quickly, the range of values of proportional which are likely to give good behaviour and, of course, the range of values which are likely to be inappropriate.

We do not discuss steady-state offsets. It is recognised that integral action is required to remove steady-state offset. Here the focus is on the role of the proportional K_p only.



$$M = K_p + \frac{K_I}{s}$$

$$G_c = \frac{GM}{1+GM}$$

Over and under actuation

Consider a zero start point and a step change in the target of r.
We are interested in the transient behaviour of the input, for example at t=0.
Given y(0)=0, the transient input is given as $K_p r$.

$$u(0) = K_p(r(0) - y(0))$$

Assuming no offset, the steady-state input is known.

$$\lim_{t\to\infty} u(t) = u_{ss} = \frac{1}{G(0)} r$$

$$\{u_{ss} > u(0)\} \quad or \quad \left\{\frac{r}{G(0)} > K_p r\right\} \quad \Rightarrow \frac{1}{G(0)} > K_p$$

$$\{u_{ss} < u(0)\} \quad or \quad \left\{\frac{r}{G(0)} < K_p r\right\} \quad \Rightarrow \frac{1}{G(0)} < K_p$$

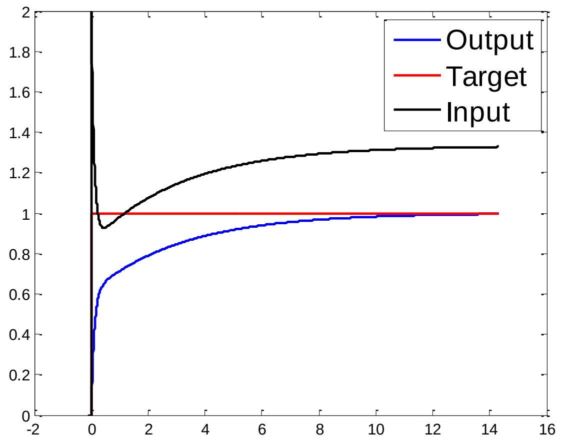
SUMMARY: The initial input is large if $K_p>(1/G(0))$ and small otherwise by which we mean it exceeds or not the expected steady-state.

1. Large transient inputs are likely to give fast transient response, compared to open-loop.
2. Small transients inputs are likely to give slow transient responses, compared to open-loop.
3. **A choice of of Kp=(1/G(0)) should set the transient input equal to the expected steady-state which means neither over or under actuation and response speed close to the open-loop.**

EXAMPLES

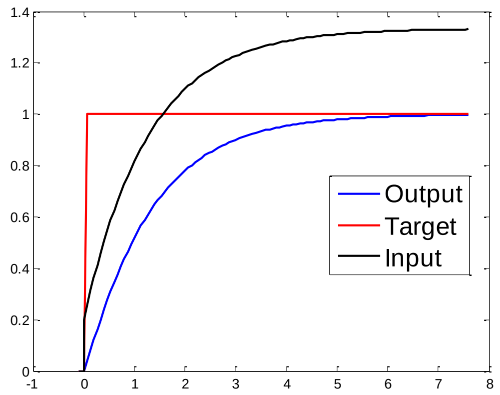
$$G = \frac{3}{s+4}; \quad M(s) = 2 + \frac{1}{s}; \quad \frac{1}{G(0)} = \frac{4}{3}$$

Over actuation expected and observed along with fast initial transient.



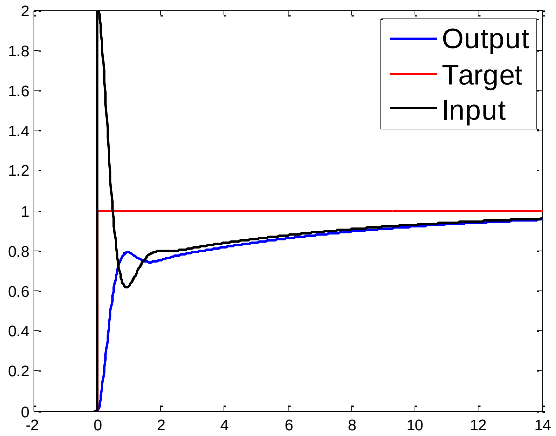
$$G = \frac{3}{s+4}; \quad M(s) = 0.2 + \frac{1}{s}; \quad \frac{1}{G(0)} = \frac{4}{3}$$

Under actuation expected and observed along with slow initial transient.



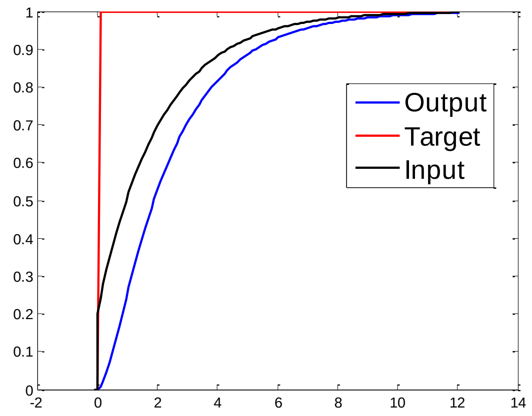
$$G = \frac{6}{s^2+5s+6}; \quad M(s) = 2 + \frac{1}{s}; \quad \frac{1}{G(0)} = 1$$

Over actuation expected and observed along with fast initial transient.



$$G = \frac{6}{s^2+5s+6}; \quad M(s) = 0.2 + \frac{1}{s}; \quad \frac{1}{G(0)} = 1$$

Under actuation expected and observed along with slow initial transient.



REMARKS: Although proportional sets the transient speed of response, there is no clear link between proportional gain and settling time. In some cases, fast transients are not beneficial as overall performance still very slow.

PROPOSAL: Logically one might argue that a good value for the proportional is to ensure the transient input matches the desired steady-state value. TRY THESE FOR YOURSELf!

$$G = \frac{3}{s+4}; \quad M(s) = \frac{4}{3} + \frac{1}{s}; \quad \frac{1}{G(0)} = \frac{4}{3}$$

$$G = \frac{6}{s^2+5s+6}; \quad M(s) = 1 + \frac{1}{s}; \quad \frac{1}{G(0)} = 1$$