Modelling and control summaries

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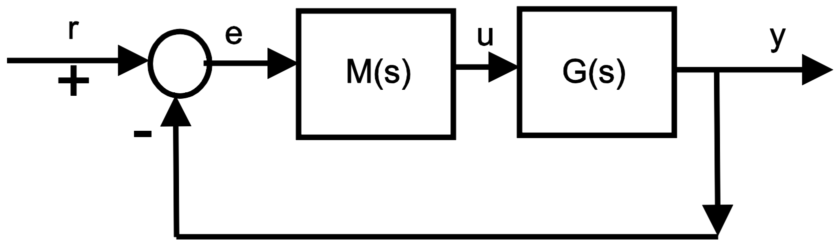
Simple feedback 3: Integral design

This brief summary gives a heuristic approach to integral design, with an overall PI compensator design. The idea is to identify, quickly, the range of values of proportional which are likely to give good behaviour and, of course, the range of values which are likely to be inappropriate. This note excludes the proportional so that readers can see the impact of using only integral action.



$$M = K_p + \frac{K_I}{s}$$

$$G_c = \frac{GM}{1+GM}$$

Role of integral

Integral is used to eliminate steady-state offset. In simple terms

$$\lim_{t\to\infty} u(t) = u_{ss} = K_I \lim_{t\to\infty} \int_0^t (r-y)dt$$

The integral, and thus u(t), will keep changing until the error between r(t) and y(t) is zero.

Assuming therefore that the asymptotic error is zero, one can define a simple formulae.

$$u_{ss} = K_I \lim_{t\to\infty} \int_0^t (r-y)dt = K_I A; \quad A = \lim_{t\to\infty} \int_0^t (r-y)dt$$

It is noted that, as u_{ss} must be fixed, therefore $K_I A$ must also be fixed!

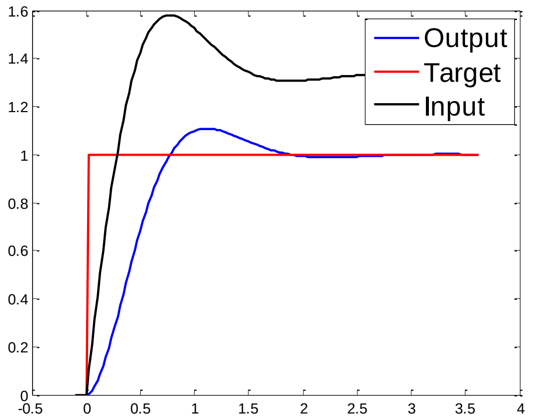
Area of error curve multiplied by K_I is a constant!

REMARK: If K_I increases, the area under the error curve should decrease ➔ faster settling time???

EXAMPLES

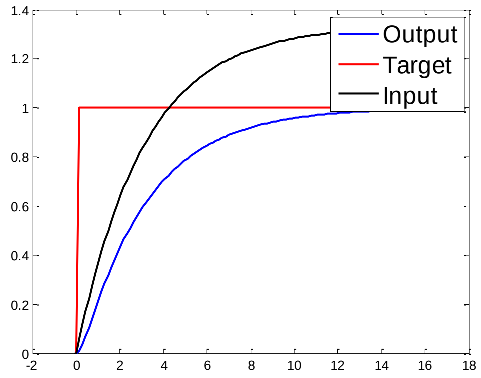
$$G = \frac{3}{s+4}; \quad M(s) = \frac{4}{s}; \quad \frac{1}{G(0)} = \frac{4}{3}$$

No asymptotic error but overshoot and oscillation.



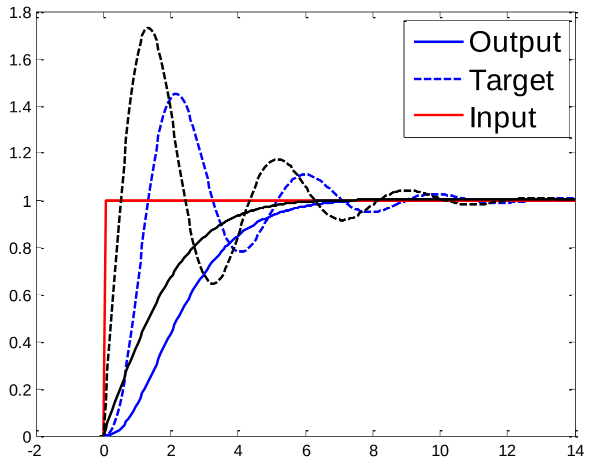
$$G = \frac{3}{s+4}; \quad M(s) = \frac{0.4}{s}; \quad \frac{1}{G(0)} = \frac{4}{3}$$

Smaller K_I results in much slower rise times and slow input changes and here, slower settling time.



$$G = \frac{6}{s^2+5s+6}; \quad M(s) = \frac{0.4}{s} \text{ or } \frac{2}{s}; \quad \frac{1}{G(0)} = 1$$

In this case, increasing K_I has led to slower convergence!



Because the error alternates in sign, the asymptotic area under the error curve may remain small even though the error is converging quite slowly. Consequently, increasing K_I, although it reduces A, may not give rise to a faster settling time (although there is a faster rise time).

SUMMARY:

Integral on its own can remove steady-state offset, however:

1. **Responses/settling can be slow especially if integral gain is low. Generally will be markedly slower than the open-loop.**
2. **Large values of integral gain tend to cause overshoot and oscillation and even potentially very slow convergence or instability.**