Modelling and control summaries

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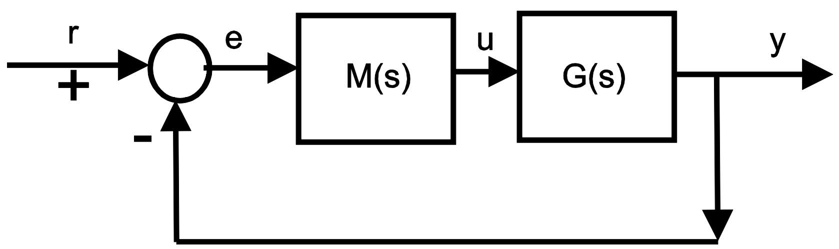
Simple feedback 4: Heuristic PI design

This brief summary gives a heuristic approach to PI design. The idea is to identify, quickly, the range of values of PI parameters which are likely to give good behaviour and, of course, the range of values which are likely to be inappropriate. The approach is heuristic and works well with simple dynamics, but is not an excuse to avoid fine tuning or more systematic design if required.



$$M = K_p + \frac{K_I}{s}$$

$$G_c = \frac{GM}{1+GM}$$

SUMMARY OF SIMPLE FEEDBACK 1-3

Proportional design

$K_p>1/G(0)$: The initial input is larger than the desired steady-state implying relatively fast transient behaviour.

$K_p<1/G(0)$: The initial input is smaller than the desired steady-state implying slow transient behaviour.

$K_p=1/G(0)$: The initial behaviour is close to open-loop dynamics.

Integral Design

$$\lim_{t\to\infty} u(t) = u_{ss} = K_I \lim_{t\to\infty} \int_0^t (r-y)dt = K_I A$$

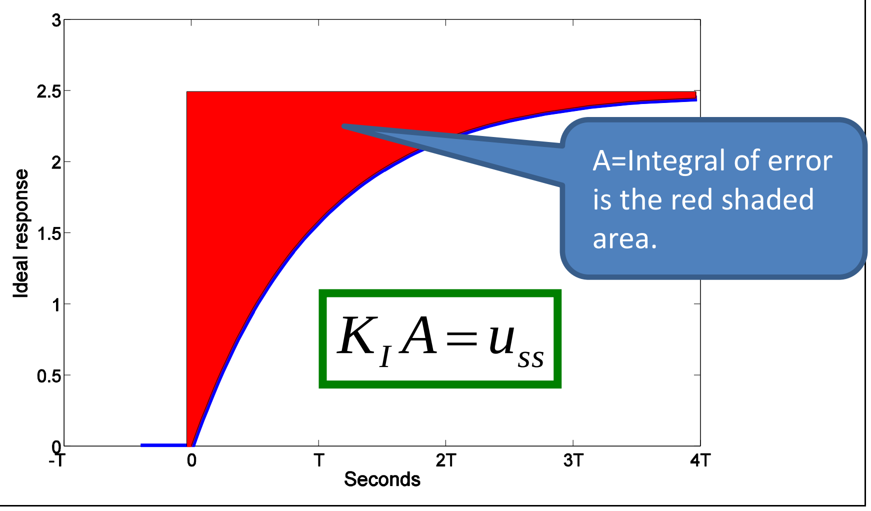
Area of error curve multiplied by K_I is a constant!

Ideally, one can control settling time by increasing/reducing K_I as this effects A directly.

Integral of error curve

If K_I increases, the red shaded area could reduce which implies faster convergence, and of course vice versa.

For a 'DESIRED ERROR' curve, one can estimate the implied A and therefore the required K_I.



Using ideal error curve

Assume the error curve follows a classic first order response with time constant T, settling a steady-state R.

The area above can be computed to be EXACTLY ***A=RT***

Moreover, the steady-state input is known to be: $u_{ss} = \frac{R}{G(0)} = K_i A$

Combining these two gives: $\frac{R}{G(0)} = K_I A = K_I RT \Rightarrow \frac{1}{TG(0)} = K_I$

HEURISTIC GUIDANCE SUMMARY: A PI that may achieve closed-loop behaviour with close to open-loop dynamics and no offset is given by

$$\left\{K_p = \frac{1}{G(0)}; \quad \frac{1}{TG(0)} = K_I\right\} \Rightarrow M(s) = \frac{1}{G(0)}\left[1 + \frac{1}{Ts}\right]$$

T is the open-loop time constant

REMARKS: This works perfectly for 1st order systems.

For 2nd order systems the response curve is much slower to move and thus has significantly larger areas for the same settling time. This means the associated integral term K_I needs to be noticeably smaller.

MAKING THE CLOSED-LOOP FASTER THAN THE OPEN-LOOP

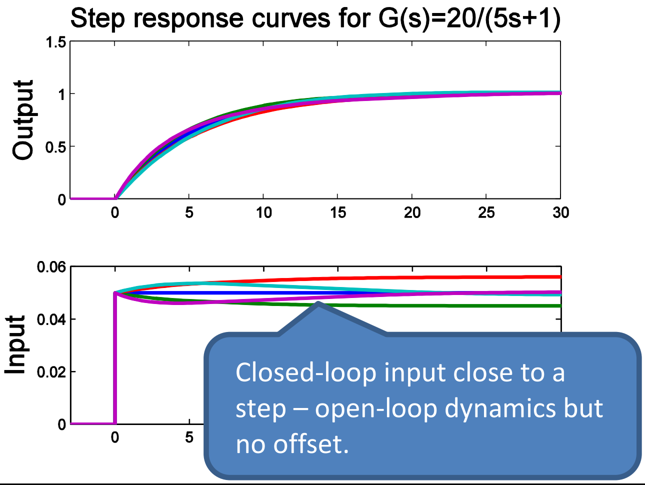
Example speed up by factor of 2:

$$M(s) = \frac{2}{G(0)}\left[1 + \frac{1}{Ts}\right]$$

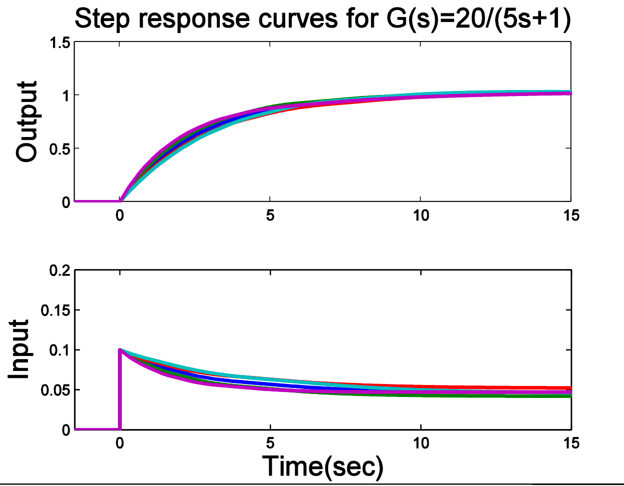
1. Larger proportional is needed to get a faster response, but the relative size is an indicator of the speed-up and/or over actuation.
2. Faster settling requires a smaller T and larger K_p. However, it will be unrealistic in general to get a settling time a lot faster than the open-loop.

EXAMPLES (parameter uncertainty included to demonstrate robustness)

Target open-loop dynamics.



With Speed-up – note initial input bigger.



SUMMARY:

Presented a heuristic PI design based on understanding of key process attributes:

- **Steady-state gain.**
- **Desired closed-loop time constant.**

Works very well with 1st order processes but gives an integral gain that this slightly high for 2nd order processes.