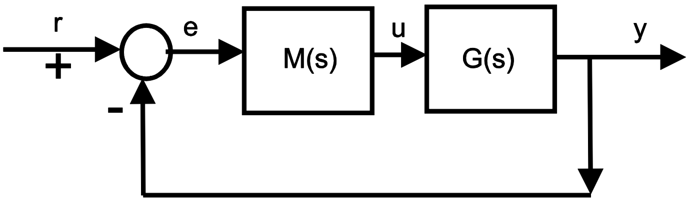
Model Predictive control for beginners *by Anthony Rossiter*

The University Of Sheffield.

Classical control 1: Introduction

Classical control techniques such as root-loci, Bode, Nyquist and indeed state-space methods such as LQR assume linear analysis is valid. This includes any discussion of robust stability margins and thus an inherent capacity to deal with some level of model uncertainty. However, all real process include constraints such as limits in absolute values and rates for actuators (inputs), desired safety limits on outputs and states, desired quality limits on outputs (linked to profit) and so forth. Whenever a system comes up against a constraint, then the overall system behaviour is highly likely to be come non-linear and therefore any linear analysis is not longer valid. Indeed, one can easily come up with examples where a linear feedback systems is supposedly very robust to parameter uncertainty, but the inclusion of a constraint causes instability. The main purpose of this note is to give some illustrations of the dangers of constraints but not to discuss possible solutions.

For this note, a simple feedback structure is assumed as follows with compensator M(s) and process G(s).



PID Compensation

PID or proportional, integral and derivative control laws are the most commonly adopted structure in industry. Their ubiquity and success is linked to the three parameters being intuitive feedback parameters and relatively simple to tune for many cases. For a typical PID compensator, M(s) has the following structure:

$$M(s) = K_p + \frac{K_I}{s} + K_d s$$

- Proportional or K_p: The magnitude of the control action is proportional to the size of the error. As the proportional is increased, the response to error becomes more aggressive leading to faster responses. If the proportional is too small, the response to error is very slow. Usually, at least for systems with simple dynamics, a proportional exists which gives the right balance between speed of response and input activity.
- Integral or K_I: Proportional alone cannot enable output tracking as, in steady-state, the output of the proportional is only non-zero if the tracking error is non-zero. Integration of the error is an obvious (also human based strategy) and simple mechanism to enable a non-zero steady-state controller output, even when the steady-state error is zero. As with proportional, K_I must not be too large or it will induce oscillation and not too small or one will get very slow convergence.
- Derivative or K_d: An obvious component to detect the rate at which the error is reducing and change the input accordingly - if the error is reducing too fast it is highly likely that the control is too aggressive and a reduction in input is needed to avoid overshoot and oscillation. However, the derivative is often selected to be zero as it has a high gain at high frequency thus can accentuate noise and lead to input chatter. Such discussions are beyond the remit of this resource.

Several tuning rules for PID are given in the literature but actually, for a typical system, one could arrive at close to 'optimum' values with very little trial and error using a simulation package.

Lead and Lag Compensation

A lag is a low gain strategy where the user wishes to improve the low frequency gain (by a factor of α relative to high frequency gain), but without detriment to transient behaviour. The corner frequencies are usually chosen well below the bandwidth (cross-over frequencies) and a typical structure is given as:

$$M(s) = K\frac{s+w}{s+{w}/{\alpha}}; \quad \alpha > 1$$

A lead has a similar structure, but in this case the aim is to improve gain in mid to high frequencies, that is to improve closed-loop bandwidth; typically the corner frequencies are set near to the desired bandwidth. The downside is a loss of low frequency gain and thus a lead may often be used in conjunction with a lag (or integral) to recover the low frequency gain.

$$M(s) = K\frac{s+{w}/{\beta}}{s+w}; \quad \beta > 1$$

Control design

Many real systems have relatively simple dynamics and there is one very important observation that is used by control practitioners and theoreticians alike.

For single input single output (SISO) systems, if the system dynamic can be well approximated by a second order (or 1st order) system which is not noticeably under-damped, then the control you can achieve from a PID compensator is usually quite close to the best you can get from optimal control or any other advanced strategy.

In other words, you need a good reason not to use a PID/classical type of approach!

In the remainder of this chapter, consideration is given to systems which do not fit into this classification and thus for which a classical design is not so straightforward and often not effective enough. It is assumed that readers are familiar with standard tools such as Bode diagrams, Nyquist diagrams and root-loci plots.