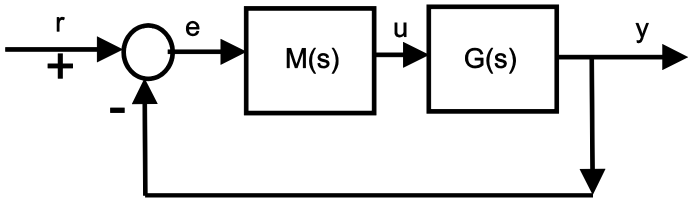
# Model Predictive control for beginners *by Anthony Rossiter*

The University Of Sheffield.

## Classical control 1.2: Non-minimum phase zeros

This section looks at the efficacy of classical control methods with systems including non-minimum phase characteristics. A simple feedback structure is assumed as follows with compensator M(s) and process G(s).



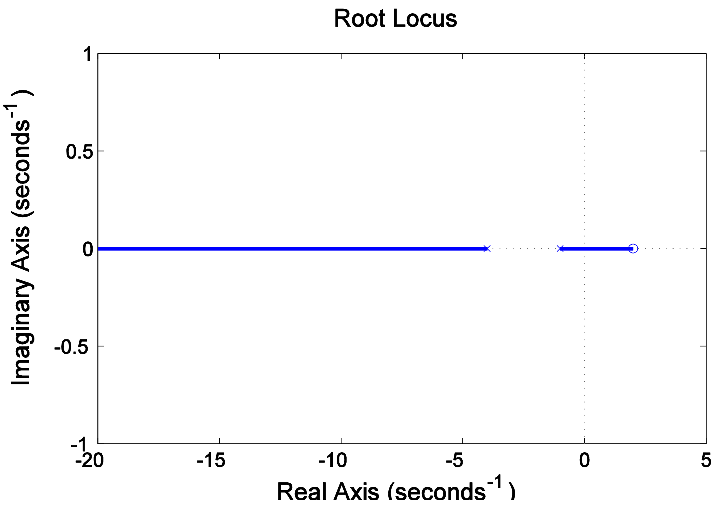

### Example 1

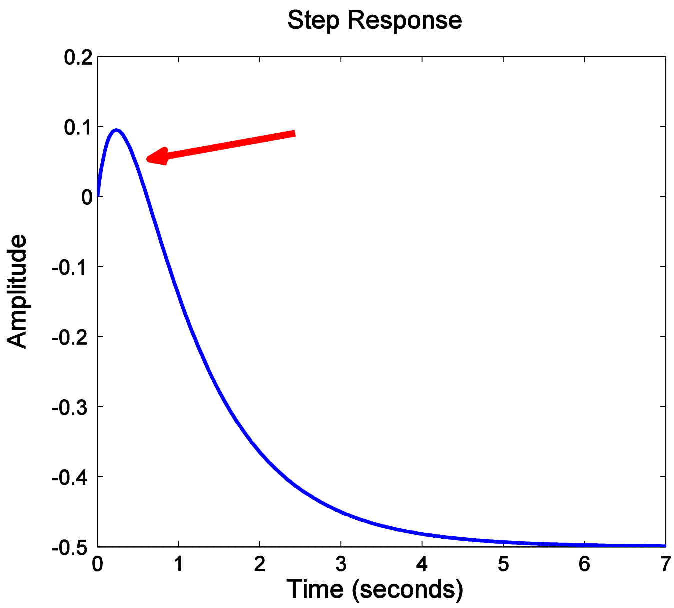
Consider the system:

$$G(s) = \frac{s-2}{(s+1)(s+4)}$$

The zero is in the RHP and therefore, by inspection from the root-loci plot, a closed-loop pole will also be in the RHP for large gains.

The open-loop step response has the characteristic non-minimum phase dip.




This means that a high gain control law can easily lead to instability because the initial response is opposite to what might be expected and thus the normal control logic can get confused; it is important to be patient (cautious or low gain) before responding to observations to ensure one does not respond to the non-minimum phase characteristic.

Here you can find a PI or PID compensator to give reasonable closed-loop behaviour. First note that due to the negative steady-state gain, it is likely to be judicious to use positive feedback. A simple PI design can then be used to form a root-loci which has all its paths in the LHP for low gains. A simple and effective compensator is something like:

$$M(s) = -K\frac{s+1}{s}$$

Root-loci or other methods can be used to find a K which gives a reasonable balance between speed of response and damping (K= 1.2 gives a pole at -1.4+0.7j).

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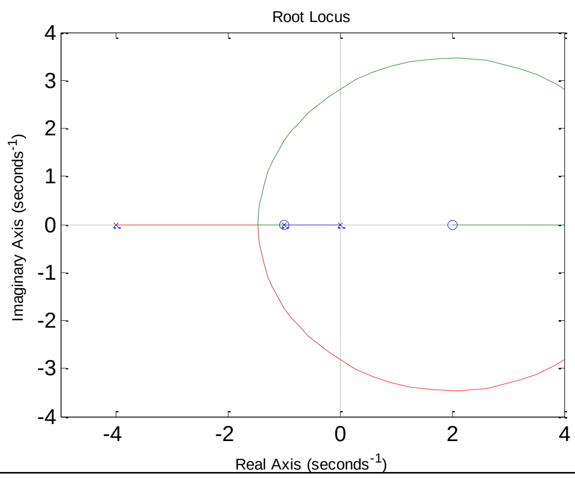
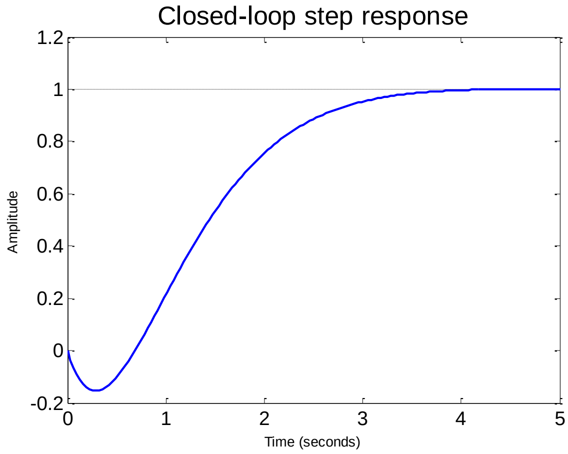
|  |  |
| **Root-loci for G(s) and M(s) for example 1** | **Closed-loop step response for G(s) and M(s)** |





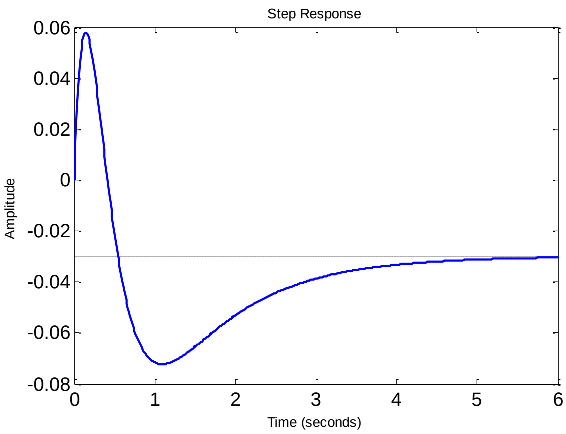
**REMARK**: In this case it is notable that an optimal control law cannot really improve on the behaviour, but this may be unsurprising given that this is a 2$^{nd}$ order model and thus PI has enough flexibility to manipulate the dynamics as required.

## Example 2

Next consider the system:

$$G(s) = \frac{(s+0.3)(s-2)}{(s+1)(s+4)(s+5)}$$

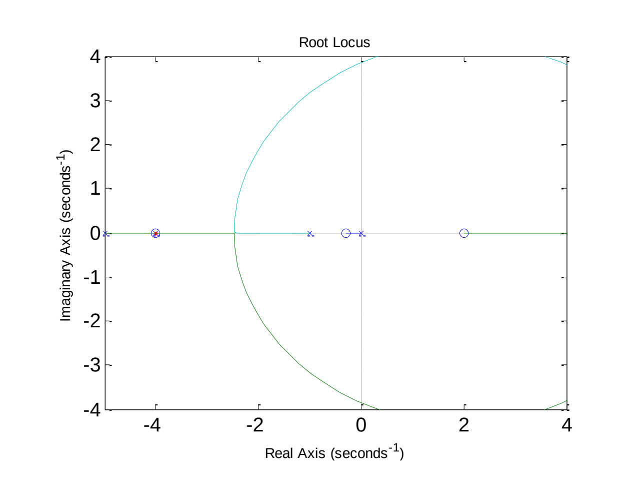
One zero is in the RHP and as for example 1 the system has a negative steady-state gain. The open-loop step response has a very marked non-minimum phase characteristic.



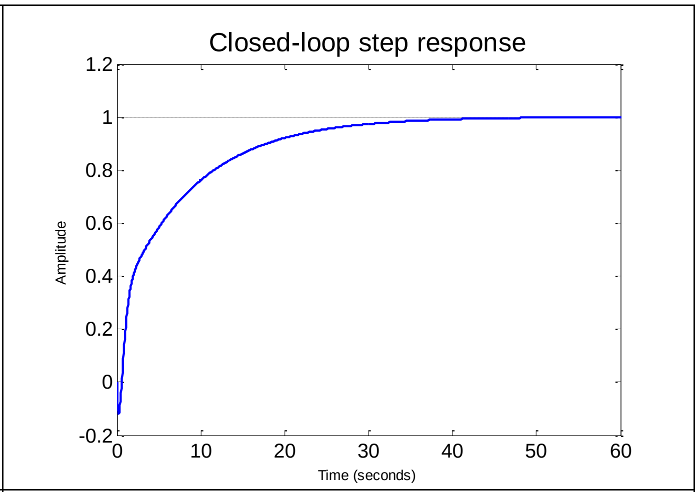

Although you can find a PI or PID compensator to give reasonable dynamics, in this case, it is less straightforward to tune and critically, much harder to get high bandwidth responses. A simple PI design can be used to form a root-loci which has all its paths in the LHP for low gains. A simple compensator is something like:

$$M(s) = -1.2\frac{s+4}{s}$$

With closed-loop poles at -4, -2.35+j,-2.35-j,-0.11). However, it is clear that the closed-loop step response is somewhat slow because the PI compensator does not have enough degrees of freedom to cater for the poorly positioned open-loop zeros and the requirement for offset free tracking.

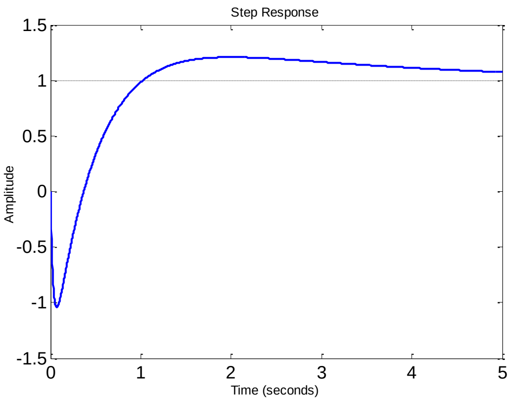



**Root-loci for G(s) and M(s) (Example 2)**




**Closed-loop step response for G(s) and M(s)**

**REMARK**: In this case an optimal control approach is able to achieve far faster convergence, although one may debate where this is really improved overall behaviour given the significant oscillations and undershoot during transients. Here, the optimal control law has, in essence, moved the input more quickly to the desired steady-state value and thus embedded the open-loop dynamics. One can reduce the undershoot and overshoot with more tuning, but at the expense of more input activity.




## SUMMARY

Classic control methods can deal with non-minimum phase characteristics to some extent, but as the dynamics becoming more challenging, an effective classical design requires more insight/expertise and may not have the flexibility to achieve all the desired objectives well.