Model Predictive control for beginners *by Anthony Rossiter*

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Classical control 1.3: delays

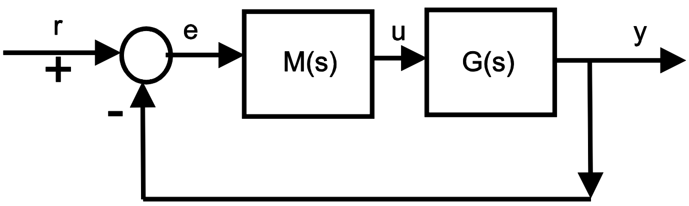
This note gives a brief review of delays and simple classical methods for dealing with delays. First however, it is important to demonstrate why delays within a process can have catastrophic effects on closed-loop behaviour. Delays can be caused by problems with measurement as some things cannot be measured instantaneously (such as blood tests in hospitals). They may also be caused by transport delays, for example a delay between a demanded actuation and the impact effecting the process; for example this would be typical where a conveyor belt is being used. For this note, we are less interested in the cause of the delay than its existence and will use a simple delay model as follows:

$G(s)$	$e^{-sT}G(s)$
Undelayed process	Process with delay of T seconds

Before reviewing the impact of delays on closed-loop behaviour, we should note that when embarking on a complicated control design, it is always worth asking whether one can change the process in order to reduce or remove the delay. Sometimes actions such as moving a sensor are possible and can given substantial benefits. It is always better to have as little delay as possible.

A reader can immediately see the dangers of delay using the simple analogy of driving a car. How would your driving be affected if you had to wait 2 seconds between observing something (in effect a 2 second measurement delay) and making a change to the controls (accelerator, brake, steering). Clearly, this delay would cause numerous accidents, dead-pedestrians, going through red-lights and so forth if driving at normal speeds. One could only avoid such incidents by driving slow enough so that you were guaranteed not to hit anything within the next 3-4 seconds, thus leaving 1-2 seconds for any required action. The key point here is that you would have to **DRIVE VERY SLOWLY** and thus sacrifice closed-loop bandwidth/performance. The larger the delay, the more performance is sacrificed.

Hereafter a simple feedback structure is assumed as follows with compensator M(s) and process G(s).



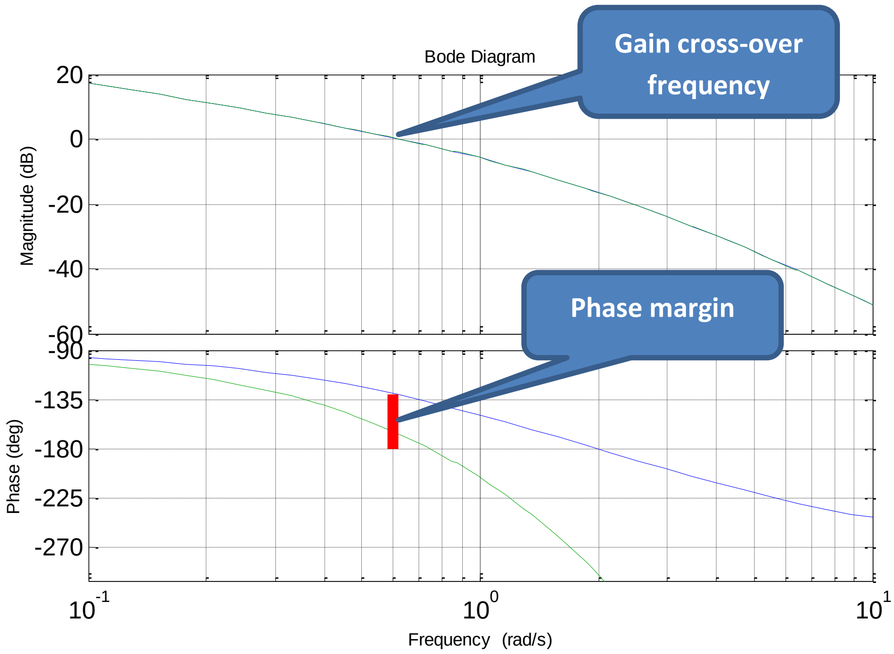
Nyquist and gain/phase margins

Nyquist diagrams are useful for illustrating the impact of delays on the gain and phase margins and thus indirectly on closed-loop behaviour. A delay acts like a phase rotation within the Nyquist diagram and specifically, rotates the diagram clockwise, thus reducing margins. The larger the delay, the more the rotation. Of key importance is the rotation near the cross-over frequencies.

$$e^{-sT} \to e^{-jwT}; \quad \angle e^{-jwT} = -wT$$

Clearly, a delay of T seconds will give a change in phase margin of roughly $w_g T$ where w_g is the gain cross over frequency. This is illustrated in the figures below which show the margins for the following two systems:

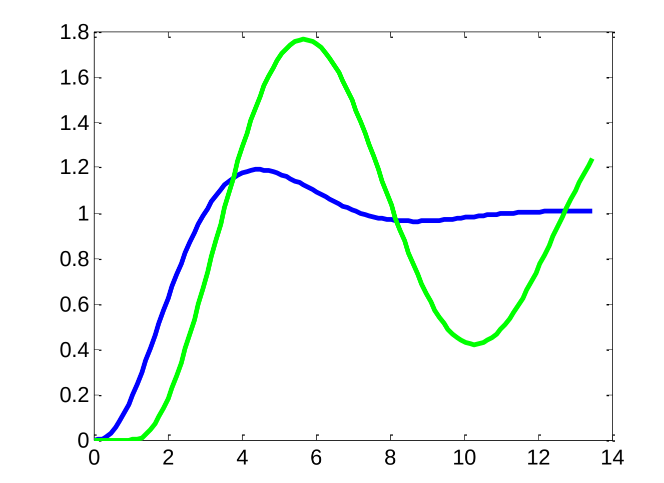
$$G(s)=\frac{3}{s(s+1)(s+4)};\quad H(s)=\frac{3e^{-s}}{s(s+1)(s+4)}$$



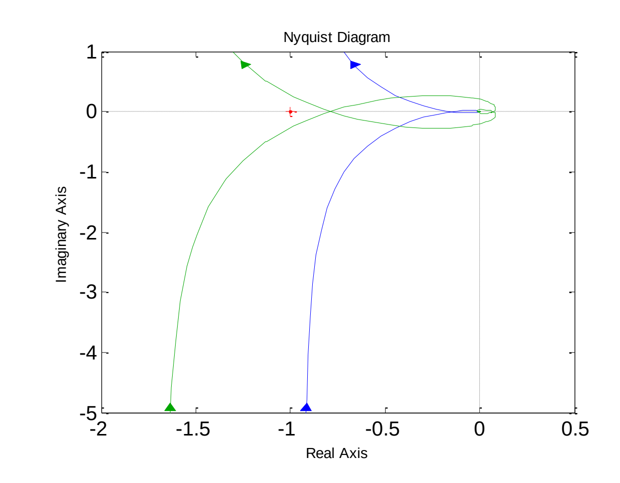
It is clear that the phase margin has dropped from about 50° to around 15° as a consequence of the delay. The gain cross-over frequency is about 0.6, and hence the loss in phase margin is about 0.6 radians or about 35°.

The gain plot is unaffected by the delay.

The closed-loop step responses and Nyquist diagrams emphasise the point that adding the delay has caused a significant degradation in performance.



Closed-loop step responses for G(s) and H(s) (with and without the delay) using unity negative feedback.

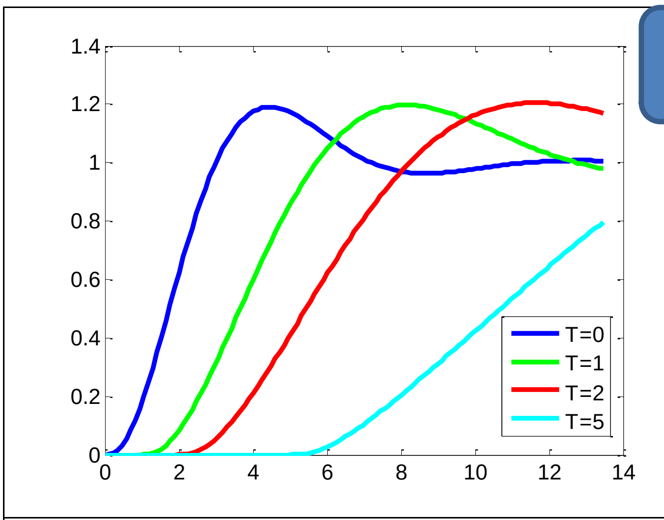


Uncompensated Nyquist diagrams for G(s) and H(s), that is with and without the delay.

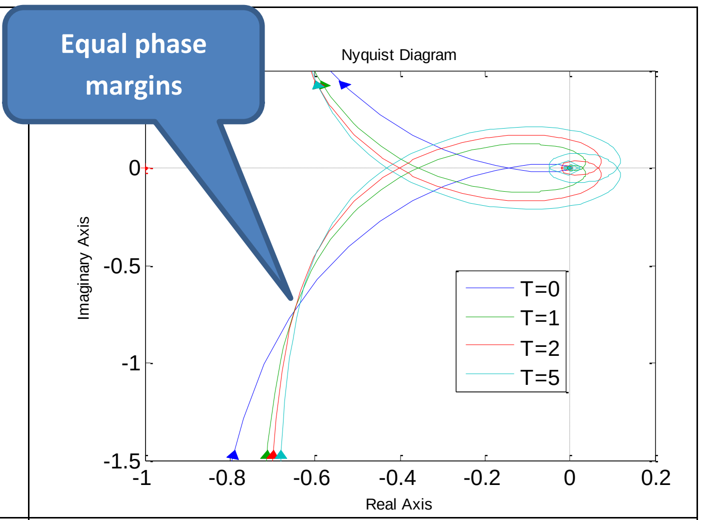
Readers may note that with a delay only a little bigger than one, the Nyquist diagram will encircle the -1 point and the system will be closed-loop unstable. Clearly, in order to regain a reasonable phase margin, similar to the undelayed case, then a significant reduction in gain is required. The figures below show the impact of delay on gain with a target of a 50° phase margin. In this case the gains required are:

$$\{T=0,K=1\},\quad \{T=1,K=0.45\},\quad \{T=2,K=0.3\},\quad \{T=5,K=0.15\}$$

It is clear therefore, that significant delay has required a significant loss in bandwidth in order to retain reasonable behaviour, limited overshoot/oscillations and good margins.



Closed-loop step responses with gain selected to give about a 50^o phase margin.



Nyquist diagrams with gain selected to give about a 50^o phase margin.

SUMMARY: Classical design techniques are often not appropriate for delaying with delays due to the significant phase lag which is implied. A normal method for dealing with phase lag is simply to reduce the gain in order to achieve positive phase margins, but in the case of delays this could require large gain reductions, especially where the delays are significant. Such a sacrifice in performance and bandwidth may not be desirable in general.