Model Predictive control for beginners *by Anthony Rossiter*

The University Of Sheffield.

Classical control 1.5: MIMO systems

Classical control techniques such as root-loci, Bode, Nyquist are targeted at SISO systems. While some extensions to the MIMO have appeared in the literature, in the main these are clumsy and awkward to use and often do not lead to systematic or satisfactory control designs. For example, one popular technique is multivariable Nyquist whereby pre- and post-compensators are used to diagonalise the process. A diagonal process can be treated as a set of SISO systems with no interaction between loops and thus normal SISO techniques can be used on each loop. [Connect input 1 with output 1 and so forth, noting that inputs and outputs are based on the pre- and post-compensated system rather than actual inputs and outputs.]

$$G = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots & g_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ g_{n1} & g_{n2} & \cdots & g_{nn} \end{bmatrix}; \quad K_{post} G K_{pre} = \begin{bmatrix} h_{11} & 0 & \cdots & 0 \\ 0 & h_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{nn} \end{bmatrix}$$

In practice, even with pre- and post-compensation, it is not possible to diagonalise a process completely, but the hope is that the off diagonal elements will be small compared to the diagonal elements.

$$G = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots & g_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ g_{n1} & g_{n2} & \cdots & g_{nn} \end{bmatrix}; \quad \begin{matrix} |g_{11}| \succ |g_{12}| + \cdots |g_{1n}| \\ \vdots \\ |g_{nn}| \succ |g_{n1}| + \cdots |g_{n,n-1}| \end{matrix}$$

In this case, one can do a design based just on the diagonal elements and this is likely to give a reasonable result, but the bigger the off diagonal elements the less applicable this approach will be. In this note we will not discuss these approaches further as:

- They have limited applicability.
- Identifying suitable pre- and post-compensation is non-trivial in general.
- Pre- and post-compensation make it harder to manage constraints on the actual inputs.

In practice, a more common practice in industry is experience of a given process whereby over time operators have learnt a control structure which works well enough for the MIMO process in question. It is of course quite possible, that the corresponding structure results in cautious control.

Summary: Stability and performance analysis for a MIMO process is non-trivial using classical techniques such as Nyquist. The consequence of this is that systematic control design is difficult and in some cases not possible with classical techniques. However, for stable processes which are near diagonal, PID compensation is often still used and is reasonably effective.

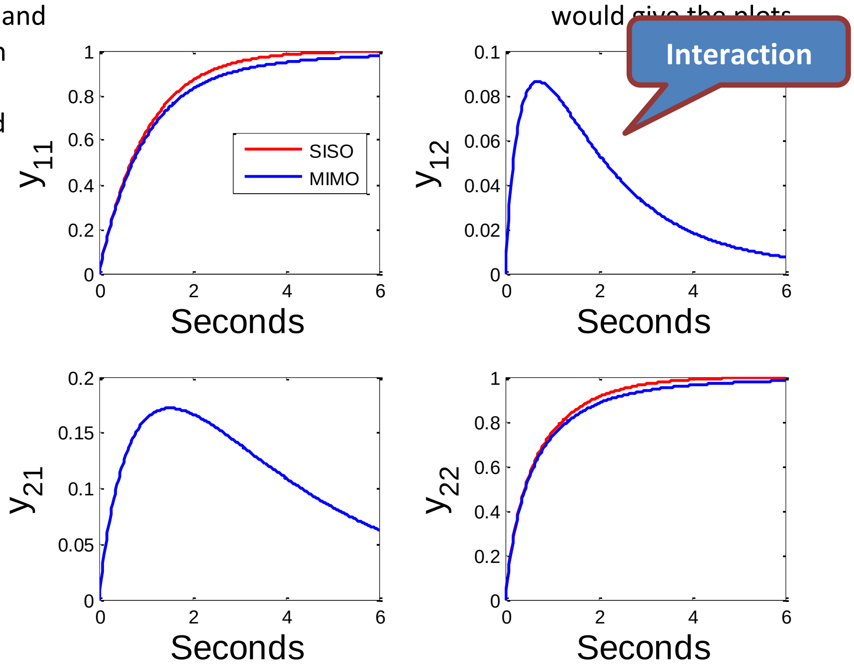
In the following we demonstrate the consequences of using conventional PI type approaches on a MIMO process which is not nearly diagonal. It is clear that while a simple approach can work at times, at other times the closed-loop behaviour is difficult to predict and significant detuning may be required to ensure stability and smooth behaviour.

Example 1 – MIMO system with mild interaction

Consider the following system which is close to diagonally dominant:

$$G = \begin{bmatrix} \dfrac{1}{(s+1)^2} & \dfrac{0.2}{s+3} \\ \dfrac{0.4}{s+0.4} & \dfrac{s+5}{(s+2)(s+3)} \end{bmatrix}; \quad K = \begin{bmatrix} \dfrac{s+1}{s} & 0 \\ 0 & \dfrac{s+2}{s} \end{bmatrix}$$

The PI design is done ignoring interaction and would give the plots appearing in red. It can be seen that when applied to the full MIMO system the diagonal outputs are almost the same and hence a SISO approach to the design has been reasonable effective. However, there is some interaction in the off diagonals, although this may be considered small.



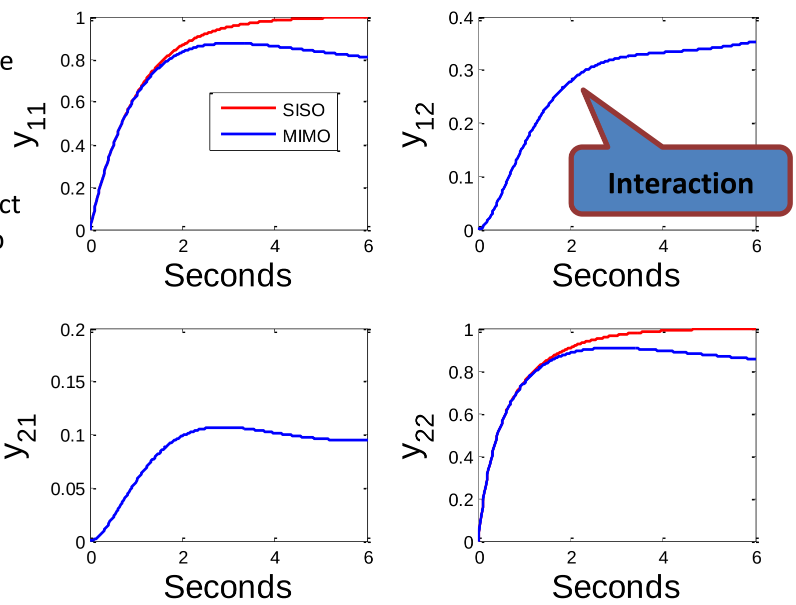
The figure shows the closed-loop output responses for step changes in each respective target (column 1 for changes in target 1 and column 2 for changes in target 2).

Example 2 – MIMO system with significant interaction

Next consider the following system which is NOT close to diagonally dominant:

$$G = \begin{bmatrix} \dfrac{1}{(s+1)^2} & \dfrac{0.5}{(s+0.2)(s+3)} \\ \dfrac{0.3}{(s+1)(s+0.4)} & \dfrac{s+5}{(s+2)(s+3)} \end{bmatrix}; \quad K = \begin{bmatrix} \dfrac{s+1}{s} & 0 \\ 0 & \dfrac{s+2}{s} \end{bmatrix}$$

The PI design is done ignoring interaction and would give the plots appearing in red. It can be seen that when applied to the full MIMO system the diagonal outputs begin almost the same but rapidly the impact from the non-diagonal elements begins to have an effect so that here the system is actually closed-loop unstable.



The figure shows the closed-loop output responses for step changes in each respective target (column 1 for changes in target 1 and column 2 for changes in target 2).