Modelling and control summaries

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Inverse Laplace 1 – introduction

TECHNIQUE FOR INVERSE LAPLACE - USE A LOOK-UP TABLE (see Laplace 4)
1. **rearrange the existing Laplace transform into the forms within the table.**
2. **inverse Laplace is then done by inspection**
This file gives some examples. [Open Laplace 4 to find the table]

EXAMPLES (Go from left to right to see working)

F(s)	STEP 1: Observation from similar format in the table	STEP 2: Put F(s) in table format	STEP 3: Find f(t)
$\frac{5}{s+2}$	$\frac{1}{s+a} \Leftrightarrow e^{-at}$	$5\left(\frac{1}{s+2}\right)$	$5e^{-2t}$
$\frac{20}{(s+1)^2+4}$	$\frac{w}{(s+a)^2+w^2} \Leftrightarrow e^{-at}\sin wt$	$10\left(\frac{2}{(s+1)^2+2^2}\right)$	$10e^{-t}\sin 2t$
$\frac{2(s+4)}{s^2+8s+20}$	$\frac{s+a}{(s+a)^2+w^2} \Leftrightarrow e^{-at}\cos wt$	$2\left(\frac{(s+4)}{(s+4)^2+2^2}\right)$	$2e^{-4t}\cos 2t$
$\frac{6}{s^2+2s}$	$\frac{1}{s+a} \Leftrightarrow e^{-at}$ & $\frac{1}{s} \Leftrightarrow 1$	$\frac{3}{s}-\frac{3}{s+2}$	$3-3e^{-2t}$
$\frac{1}{(s+1)(s+2)}$	$\frac{1}{s+a} \Leftrightarrow e^{-at}$	$\frac{1}{s+1}-\frac{1}{s+2}$	$e^{-t}-e^{-2t}$

ENABLING SKILLS - PARTIAL FRACTIONS

1. Identify key forms in the table that contribute to the current transform.
2. Use a partial fractions technique to separate the transform into these forms.
3. To do partial fractions students must be competent at factorising polynomials.
4. For partial fractions we use roots of the denominator (**these are denoted as poles**).

SUMMARY of KEY STEPS	
Factorise the denominator. The chosen factors should match the poles of forms in the lookup table, that is: s^n, (s+a) or $[(s+a)^2+w^2]$	$F(s)=\frac{n(s)}{d(s)} \Rightarrow d(s)=r_1(s)r_2(s)\cdots r_n(s)$
Re-write the Laplace transform in terms of its partial fractions using the given factors. **Make sure the format of $C_i(s)$ is such that each term matches a form in the look-up table. This is critical for the final step.**	$F(s)=\frac{Q(s)}{P(s)}=\frac{C_1(s)}{r_1(s)}+\frac{C_2(s)}{r_2(s)}+\ldots+\frac{C_n(s)}{r_n(s)}$
Use the look-up table to find f(t)	$L^{-1}[C_i/r_i]$ of each term should be by inspection.

Examples

$F(s)=\dfrac{4}{s^2+3s+2}$	STEP 1: The poles are at -1 and -2 that is $s^2+3s+2=(s+1)(s+2)$ STEP 2: Write the expected expansion as: F = A/(s+1) + B/(s+2) and hence determine residues A, B. STEP 3: By inspection $f(t)=Ae^{-t}+Be^{-2t}$.
$F(s)=\dfrac{4(s+1)}{s^2+4s}$	STEP 1: The poles are at 0 and -4 that is $s^2+4s=s(s+4)$ STEP 2: Write the expected expansion as: F = A/s + B/(s+4) and hence determine residues A, B. STEP 3: By inspection $f(t)=A+Be^{-4t}$.
$F(s)=\dfrac{2}{s^3+4s^2+3s}$	STEP 1: The poles are at 0, -1 and -3 that is $s^3+4s^2+3s=s(s+3)(s+1)$ STEP 2: Write the expected expansion as: F = A/s + B/(s+3) + C/(s+1) and hence determine residues A, B, C. STEP 3: By inspection $f(t)=A+Be^{-3t}+Ce^{-t}$.
$F(s)=\dfrac{5}{s^3+10s^2+25s}$	STEP 1: The poles are at 0, -5, -5, that is $s^3+10s^2+25s=s(s+5)(s+5)$ **Note the repeated root which impacts on the expansion!** STEP 2: Write the expected expansion as: F = A/s + B/$(s+5)^2$ + C/(s+5) and hence determine residues A, B, C. STEP 3: By inspection $f(t)=A+Bte^{-5t}+Ce^{-5t}$.
$F(s)=\dfrac{5(s+6)}{s^3+25s^2}$	STEP 1: The poles are at 0, 0, -25 that is $s^3+25s^2=s^2(s+25)$ **Note the repeated root which impacts on the expansion!** STEP 2: Write the expected expansion as: F = A/s^2 + B/s + C/(s+25) and hence determine residues A, B, C. STEP 3: By inspection $f(t)=At+B+Ce^{-25t}$.
$F(s)=\dfrac{23}{(s+1)(s^2+2s+5)}$	STEP 1: The poles are at -1, -1±2j that is $s^3+4s^2+3s=s((s+1)^2+2^2)$ STEP 2: Write the expected expansion as: $F = A/(s+1) + 2*B/((s+1)^2+2^2) + C(s+1)/((s+1)^2+2^2)$ and hence determine residues A, B, C. **NOTE care was taken to ensure the partial fraction forms matched those in the Laplace look-up table.** STEP 3: By inspection $f(t)=Ae^{-t}+e^{-t}[B\sin 2t+C\cos 2t]$

TESTING YOUR SOLUTIONS WITH MATLAB: Students who are not sure if they have computed the residues correctly can use Laplace tools to check their own work.
Two screen dump examples are given here.