Modelling and control summaries

by Anthony Rossiter

The University Of Sheffield.

Inverse Laplace 3 – cover-up rule

TECHNIQUE FOR INVERSE LAPLACE - USE A LOOK-UP TABLE (see Laplace 4)
1. **rearrange the existing Laplace transform into the forms within the table.** 2. **inverse Laplace is then done by inspection** **A key part of this is the partial fraction expansion so students should be skilled in doing this quickly where possible. The cover-up rule gives some residues by inspection.**

SUMMARY of COVER-UP ASSUMPTIONS	
Re-write the Laplace transform in terms of its partial fractions by factorising the denominator.	$F(s)=\frac{Q(s)}{P(s)}=\frac{C_1(s)}{r_1(s)}+\frac{C_2(s)}{r_2(s)}+\ldots+\frac{C_n(s)}{r_n(s)}$
If $r_i(s)$ is a simple factor of the form (s+a) then residue $C_i(s)$ is known to be a constant (for now, ignoring the case of repeated roots).	If $r_i(s)$ is a simple factor of the form (s+a) then one can use the cover-up rule to determine C_i by inspection.

COVER-UP ILLUSTRATION – ASSUME SIMPLE FACTORS OF FORM (s+a) [No repeated root]	
Multiply F(s) by a single denominator factor r_1 =(s+a) This has a root at s=–a.	$\frac{Q(s)}{P(s)}r_1(s)=\frac{C_1(s)r_1(s)}{r_1(s)}+\frac{C_2(s)r_1(s)}{r_2(s)}+\ldots+\frac{C_n(s)r_1(s)}{r_n(s)}$ $\frac{Q(s)}{r_2(s)r_3(s)\cdots r_n(s)}=C_1(s)+\frac{C_2(s)r_1(s)}{r_2(s)}+\ldots+\frac{C_n(s)r_1(s)}{r_n(s)}$
Substitute s=-a into both sides and note that anything with $r_1(s)$ in the numerator will give zero as by definition $r_1(-a)=0$.	$\frac{Q(-a)}{r_2(-a)r_3(-a)\cdots r_n(-a)}=C_1(-a)+\underbrace{\frac{C_2(-a)r_1(-a)}{r_2(-a)}+\ldots+\frac{C_n(-a)r_1(-a)}{r_n(-a)}}_{=0}$ $\Rightarrow\quad\frac{Q(-a)}{r_2(-a)r_3(-a)\cdots r_n(-a)}=C_1(-a)$
Hence the residue is obtained by substituting s=-a into F(s) and evaluating while covering-up or neglecting the factor (s+a) in the denominator. ASSUMES that (s+a) is not a repeated root.	

Examples

$$F(s)=\frac{4}{(s+1)(s+2)}=\frac{C_1}{s+1}+\frac{C_2}{s+2}$$

$$C_1=\frac{4}{\quad(-1+2)}=4;\quad C_2=\frac{4}{(-2+1)\quad}=-4$$

Cover the s+1 | Put s=-1 | Put s=-2 | Cover s+2

Find C_1 by covering up the factor (s+1) in F(s) and substituting in s=-1.

Find C_2 by covering up the factor (s+2) in F(s) and substituting in s=-2.

$$F(s)=\frac{10}{s(s+1)(s+2)}=\frac{C_1}{s}+\frac{C_2}{s+1}+\frac{C_3}{s+2}$$

Find C_1 by covering up the factor (s) in F(s) and substituting in s=0.
Find C_2 by covering up the factor (s+1) in F (s) and substituting in s=-1.
Find C_3 by covering up the factor (s+2) in F(s) and substituting in s=-2.

$$C_1=\frac{10}{\quad(0+1)(0+2)}=5;\quad C_2=\frac{10}{-1\quad(-1+2)}=-10;\quad C_3=\frac{10}{-2(-2+1)\quad}=5$$

Cover the s | Put s=0 | Cover the s+1 | Put s=-1 | Put s=-2 | Cover the s+2

TESTING YOUR SOLUTIONS WITH MATLAB: Students who are not sure if they have computed the residues correctly can use Laplace tools to check their own work as in Inverse Laplace 1.

A more direct root is to use the function residue.m as illustrated here for this example.

$$F(s)=\frac{10}{s(s+2)(s+1)}$$

$$F(s)=\frac{10}{s^3+3s^2+2s+0}$$

Denominator coefficients are 1, 3, 2 and 0.