Modelling and control summaries

The University Of Sheffield.

by Anthony Rossiter

Inverse Laplace 8 – partial fractions & MATLAB

NUMERICAL TECHNIQUES FOR SOLVING INVERSE LAPLACE USING MATLAB

Many real problems have awkward numbers and should not be tackled on pen and paper. MATLAB will do partial fractions for you using residue.m and give numerical answers.

WARNING: You need to know how to interpret the resulting solutions. Residue does not give the answers you expect for sinusoidal components.

SUMMARY of PARTIAL FRACTION EXPANSION and ASSUMPTIONS

Re-write the Laplace transform in terms of its partial fractions where $r_i(s)$ are **ONLY** simple and distinct factors of the form (s+a) so that the residues C_i are known to be constants.

$$F(s)=\frac{Q(s)}{P(s)}=\frac{C_1}{r_1(s)}+\frac{C_2}{r_2(s)}+\ldots+\frac{C_n}{r_n(s)}$$

Warning: Residue.m does not distinguish between real and complex poles and will give complex residues for complex poles.

$$P=\frac{w}{s^2+w^2}=\frac{A}{s+jw}+\frac{B}{s-jw}$$

$$N=\frac{as+b}{(s+a)^2+w^2}=\frac{A}{s+a+jw}+\frac{B}{s+a-jw}$$

BASIC COMMAND

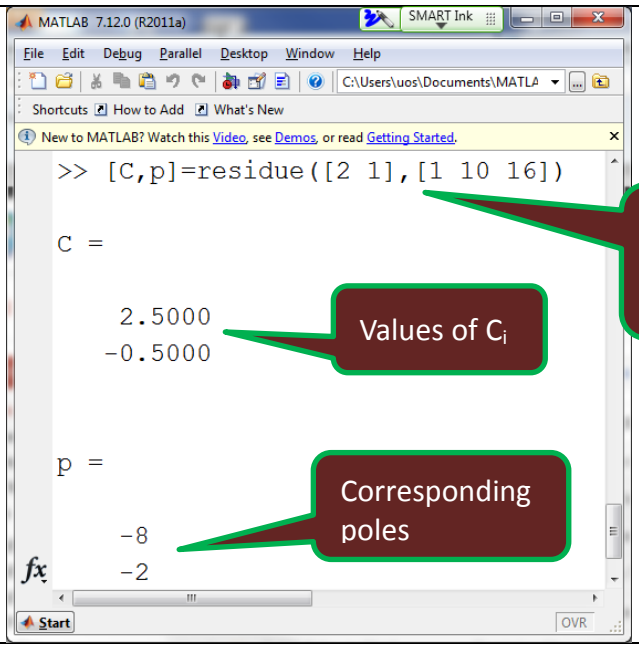
[residues, poles] = residue(num,den)

- Values of C_i
- Corresponding poles
- Vectors containing coefficients of numerator and denominator.

EXAMPLE 1

$$F=\frac{2s+1}{s^2+10s+16}$$

$$F=\frac{2.5}{s+8}-\frac{0.5}{s+2}$$

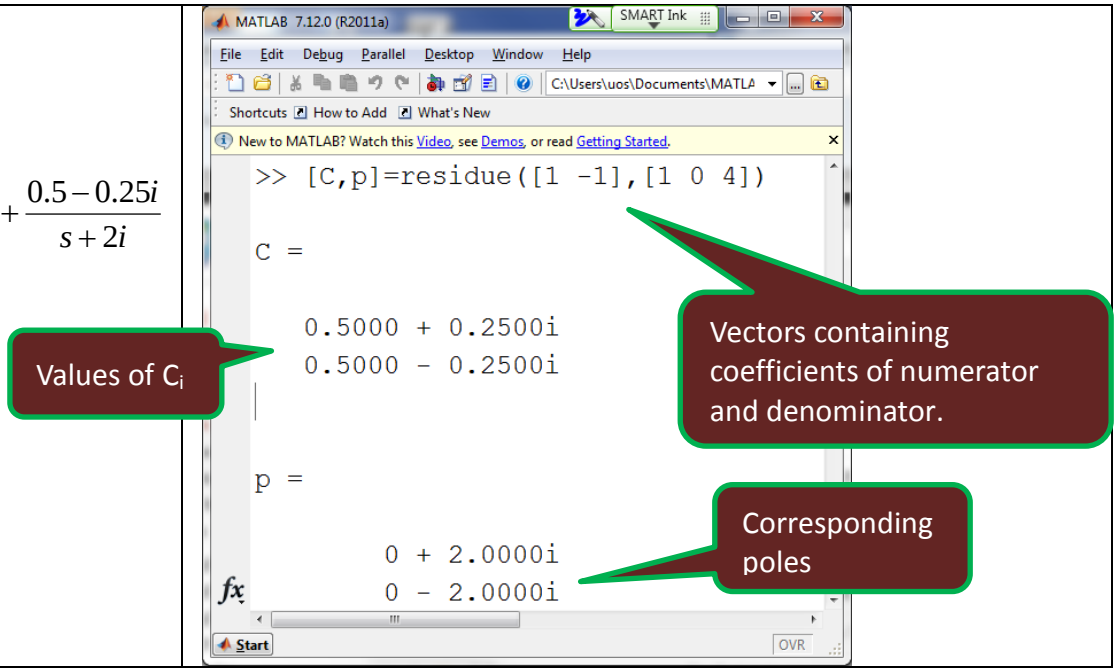


- Vectors containing coefficients of numerator and denominator.
- Values of C_i
- Corresponding poles

EXAMPLE 2

$$F = \frac{s-1}{s^2+4}$$

$$F = \frac{0.5+0.25i}{s-2i} + \frac{0.5-0.25i}{s+2i}$$



REPEATED ROOTS – THESE SHOULD BE CONSIDERED WITH CARE

Where a root, say r_1, is repeated, this will appear several times in the partial fraction expansion and have a residue associated to each different power. When using residue.m, it is important to know which residue goes with which power.

$$F(s) = \frac{Q(s)}{P(s)} = \frac{C_1}{r_1} + \frac{C_2}{r_1^2} + \ldots + \frac{C_h}{r_1^h} + \cdots + \frac{C_n}{r_n}$$

EXAMPLE 3

$$F = \frac{s+4}{s^4+s^3-3s^2-5s-2}$$

$$F = \frac{-1}{(s+1)^3} + \frac{-0.667}{(s+1)^2} + \frac{-0.222}{s+1} + \frac{0.222}{s+2}$$