

# Modelling and control summaries



by Anthony Rossiter

## Inverse Laplace 8 – partial fractions & MATLAB

### NUMERICAL TECHNIQUES FOR SOLVING INVERSE LAPLACE USING MATLAB

Many real problems have awkward numbers and should not be tackled on pen and paper.

MATLAB will do partial fractions for you using **residue.m** and give numerical answers.

**WARNING:** You need to know how to interpret the resulting solutions. Residue does not give the answers you expect for sinusoidal components.

### SUMMARY of PARTIAL FRACTION EXPANSION and ASSUMPTIONS

Re-write the Laplace transform in terms of its partial fractions where  $r_i(s)$  are **ONLY** simple and distinct factors of the form  $(s+a)$  so that the residues  $C_i$  are known to be constants.

$$F(s) = \frac{Q(s)}{P(s)} = \frac{C_1}{r_1(s)} + \frac{C_2}{r_2(s)} + \dots + \frac{C_n}{r_n(s)}$$

**Warning:** Residue.m does not distinguish between real and complex poles and will give complex residues for complex poles.

$$P = \frac{w}{s^2 + w^2} = \frac{A}{s + jw} + \frac{B}{s - jw}$$

$$N = \frac{as + b}{(s + a)^2 + w^2} = \frac{A}{s + a + jw} + \frac{B}{s + a - jw}$$

### BASIC COMMAND

**[residues, poles] = residue(num,den)**

Values of  $C_i$

Corresponding poles

Vectors containing coefficients of numerator and denominator.

### EXAMPLE 1

$$F = \frac{2s + 1}{s^2 + 10s + 16}$$

$$F = \frac{2.5}{s + 8} - \frac{0.5}{s + 2}$$

```

MATLAB 7.12.0 (R2011a)
File Edit Debug Parallel Desktop Window Help
C:\Users\uos\Documents\MATLA
Shortcuts How to Add What's New
New to MATLAB? Watch this Video, see Demos, or read Getting Started.
>> [C,p]=residue([2 1],[1 10 16])

C =

    2.5000
   -0.5000

p =

    -8
    -2
    
```

Values of  $C_i$

Corresponding poles

Vectors containing coefficients of numerator and denominator.

### EXAMPLE 2

$$F = \frac{s-1}{s^2+4}$$

$$F = \frac{0.5+0.25i}{s-2i} + \frac{0.5-0.25i}{s+2i}$$

Values of  $C_i$

```

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Shortcuts How to Add What's New
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>> [C,p]=residue([1 -1],[1 0 4])

C =

    0.5000 + 0.2500i
    0.5000 - 0.2500i

p =

    0 + 2.0000i
    0 - 2.0000i
  
```

Vectors containing coefficients of numerator and denominator.

Corresponding poles

### REPEATED ROOTS – THESE SHOULD BE CONSIDERED WITH CARE

Where a root, say  $r_1$ , is repeated, this will appear several times in the partial fraction expansion and have a residue associated to each different power. When using residue.m, it is important to know which residue goes with which power.

$$F(s) = \frac{Q(s)}{P(s)} = \frac{C_1}{r_1} + \frac{C_2}{r_1^2} + \dots + \frac{C_h}{r_1^h} + \dots + \frac{C_n}{r_n}$$

### EXAMPLE 3

$$F = \frac{s+4}{s^4+s^3-3s^2-5s-2}$$

$$F = \frac{-1}{(s+1)^3} + \frac{-0.667}{(s+1)^2} + \frac{-0.222}{s+1} + \frac{0.222}{s+2}$$

Values of  $C_i$

Corresponding poles – note  $(s+1)$  is repeated 3 times

```

MATLAB R2014a
FILE VARIABLE CODE SIMULINK ENVIRONMENT RESOURCES
C:\Users\uos\Documents\MATLAB
New to MATLAB? Watch this Video, see Examples, or read Getting Started.
>> [C,p]=residue([1 4],[1 1 -3 -5 -2])

C =

    0.2222
   -0.2222
   -0.6667
   -1.0000

p =

    2.0000
   -1.0000
   -1.0000
   -1.0000
  
```

Vectors containing coefficients of numerator and denominator.

Goes with  $(s+1)^2$

Goes with  $(s+1)^3$