Modelling and control summaries

by Anthony Rossiter

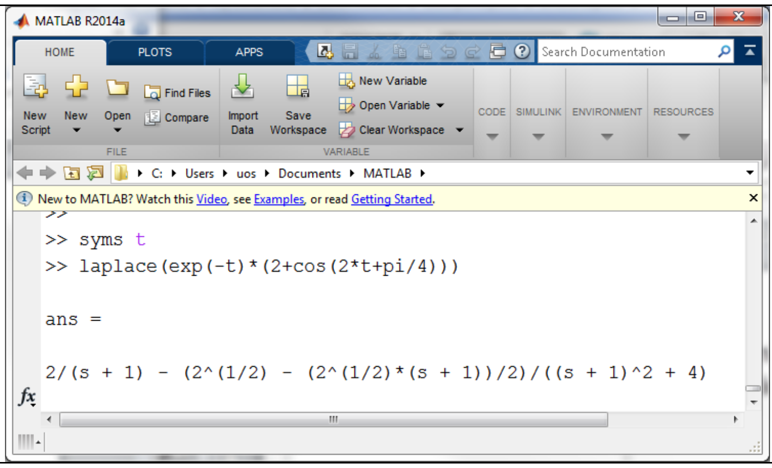
Laplace and inverse Laplace – tutorial sheet

Use a table of Laplace transforms to determine the Laplace transforms of the following functions:
Use laplace.m to test your answers.

$$f(t) = 5e^{-2t}; \qquad g(t) = 3 + \cos(2t);$$

$$h(t) = te^{-3t}; \quad m(t) = e^{-4t}\sin(3t);$$

$$n(t) = e^{-t}(2 + \cos(2t + \frac{\pi}{4}))$$



1. Find the Laplace transform of the following signals:
   $e^{-5t}; \quad \cos(0.4t); \quad te^{0.1t}; \quad e^{-10t}\sin(6t); \quad e^{-10t}\sin(6t + \pi/2); \quad e^{-2t} + e^{-3t}\cos(0.5t)$
2. Use partial fractions, a lookup table and inverse Laplace to find the underlying signals with the following transforms.
   $\frac{1}{s-0.3}; \quad \frac{-4}{s^2}; \quad \frac{1}{(s+5)^2}; \quad \frac{6}{s^2+7s+12}; \quad \frac{9}{s^2-4s+12}; \quad \frac{12}{s(s^2+7s+12)}; \quad \frac{s+4}{s^2+8s+12};$
3. What is the final value for signals with the following transforms?
   $\frac{1}{s+0.25}; \quad \frac{4}{s}; \quad \frac{15}{(s+5)^2}; \quad \frac{6}{s^2+5s+4}; \quad \frac{9}{s^2-4s+8}; \quad \frac{2}{s(s^2+5s+4)}; \quad \frac{s+1}{s^2+3s+12};$
4. Which of the following transforms has the fastest settling time? What are the settling times to within 5% of steady-state?
   $\frac{1}{s+0.25}; \quad \frac{15}{(s+5)^2}; \quad \frac{6}{s^2+5s+4}; \quad \frac{9}{s^2+6s+8}; \quad \frac{2}{s(s^2+5s+4)}; \quad \frac{s+1}{s^2+7s+12};$
5. Sketch the poles and zeros of the following transforms on an Argand diagram. By marking the LHP and RHP clearly, hence determine which represent stable and unstable behaviour.
   $\frac{1}{s+0.25}; \quad \frac{4}{s}; \quad \frac{15}{(s-5)^2}; \quad \frac{6(s-4)}{s^2+5s+4}; \quad \frac{9(s+1)}{s^2-6s+8}; \quad \frac{2}{s(s^2+5s-4)}; \quad \frac{s+0.2}{s^2+7s+12};$

6. Use the final value theorem to find the final value for signals with the following Laplace transforms. Use MATLAB to validate your answers. Also validate your answers by forming the partial fractions and computing the underlying time domain signal explicitly.

$$\frac{-0.3}{s+0.25}; \quad \frac{15}{s(s+5)}; \quad \frac{6}{5s^2+4s}; \quad \frac{9}{4s^3+8s^2}; \quad \frac{9(s+2)}{4s^3+8s^2+4s}; \quad \frac{s-1}{s^2-s-2}$$

7. Prove that a polynomial with only real LHP roots must have coefficients all the same sign. Is a polynomial having all coefficients the same sign enough to ensure that all the roots are in the LHP?

8. Determine the Laplace transforms of the following:

$$f(t) = 5e^{-2(t-3)}; \quad \{f(t) = 0, t < 3\}$$

$$m(t) = e^{-4(t-2)}\sin(3(t-2)); \quad \{m(t) = 0, t < 2\}$$

9. Determine the underlying signal

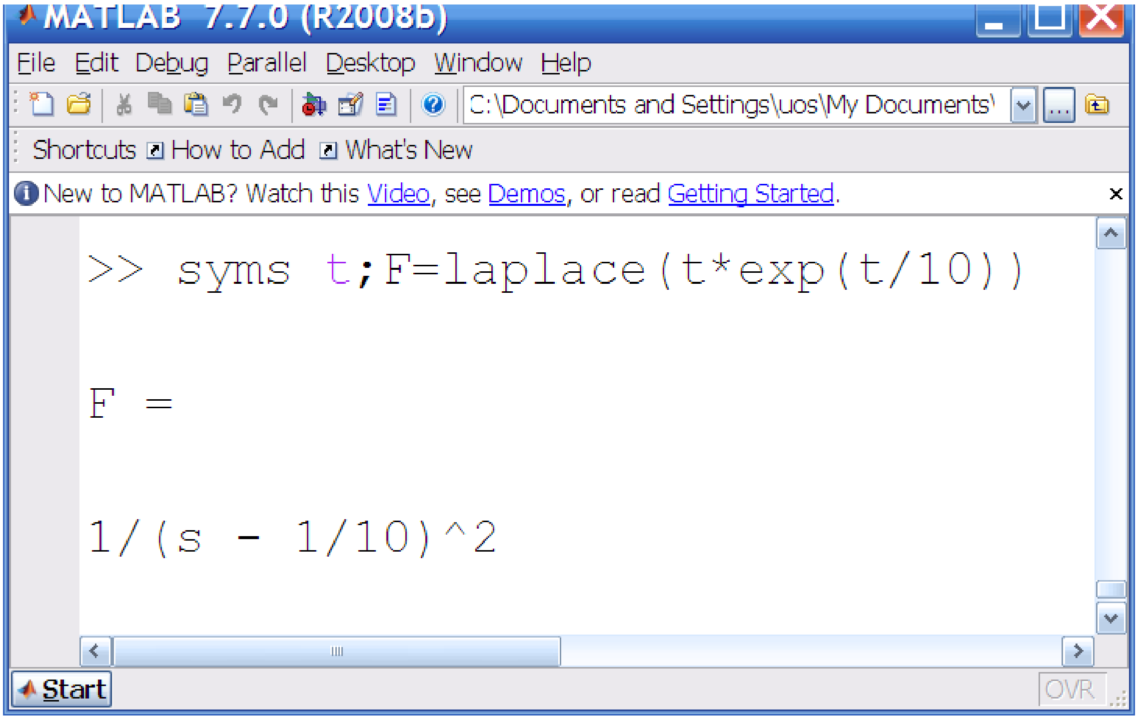
$$Y(s) = e^{-5s}\frac{3s+3}{s^2+2s+5}$$

Outline Answers – students are expected to self validate using software tools just as you would in a job

1. Find the Laplace transform of the following signals:

$$e^{-5t}; \quad \cos(0.4t); \quad te^{0.1t}; \quad e^{-10t}\sin(6t); \quad e^{-10t}\sin(6t+\pi/2); \quad e^{-2t}+e^{-3t}\cos(0.5t)$$

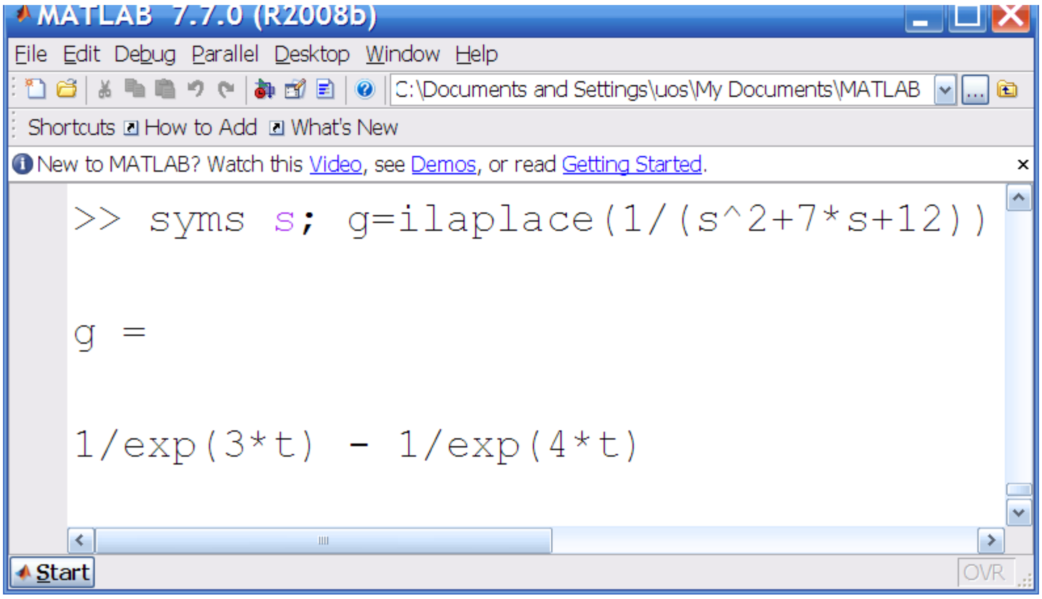
Students should use MATLAB to check their working here, e.g. try the command:



2. Use partial fractions, a lookup table and inverse Laplace to find the underlying signals with the following transforms.

$$\frac{1}{s-0.3}; \quad \frac{-4}{s^2}; \quad \frac{1}{(s+5)^2}; \quad \frac{6}{s^2+7s+12}; \quad \frac{9}{s^2-4s+12}; \quad \frac{12}{s(s^2+7s+12)}; \quad \frac{s+4}{s^2+8s+12};$$

Students should use MATAB to check their working, e.g.



3. What is the final value for signals with the following transforms?

$$\frac{1}{s+0.25};\quad \frac{4}{s};\quad \frac{15}{(s+5)^2};\quad \frac{6}{s^2+5s+4};\quad \frac{9}{s^2-4s+8};\quad \frac{2}{s(s^2+5s+4)};\quad \frac{s+1}{s^2+3s+12};$$

Use the FVT but note that: (i) there is no final value if the signal is divergent which is the case for 5^{th} (obvious from negative sign) and (ii) for convergent signals, the final value must be zero if there is no integrator. Hence only 2^{nd} and 6^{th} have a non-zero values which must be 4 and 0.5 respectively.

4. Which of the following transforms has the fastest settling time? What are the settling times to within 5% of steady-state?

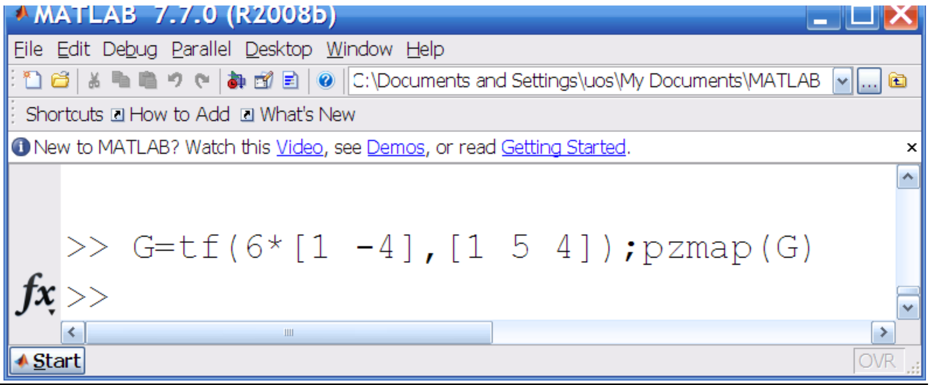
$$\frac{1}{s+0.25};\quad \frac{15}{(s+5)^2};\quad \frac{6}{s^2+5s+4};\quad \frac{9}{s^2+6s+8};\quad \frac{2}{s(s^2+5s+4)};\quad \frac{s+1}{s^2+7s+12};$$

Time constants are negative inverses of poles. One can estimate time to 5% error as approximately three times slowest time constant (exact for 1^{st} order but no strict generalistion when many poles due to uncertainty about partial fractions). Time constant is the negative inverse of the pole. So pole at -0.25 gives T=4, etc.

5. Sketch the poles and zeros of the following transforms on an Argand diagram. By marking the LHP and RHP clearly, hence determine which represent stable and unstable behaviour.

$$\frac{1}{s+0.25};\quad \frac{4}{s};\quad \frac{15}{(s-5)^2};\quad \frac{6(s-4)}{s^2+5s+4};\quad \frac{9(s+1)}{s^2-6s+8};\quad \frac{2}{s(s^2+5s-4)};\quad \frac{s+0.2}{s^2+7s+12};$$

Students should use MATLAB to check their working for this, for example, doing 4^{th} as follows will produce a figure with poles marked in 'x' and zeros in 'o':



Systems are stable if and only if all the poles are in the LHP - the origin is counted as being in the LHP. The positions of the zeros do not affect stability.

6.

$$\frac{-0.3}{s+0.25}\to 0\quad \frac{15}{s(s+5)}\to 3;\quad \frac{6}{5s^2+4s}\to 1.5;\quad \frac{9}{4s^3+8s^2}\to\infty;\quad \frac{9(s+2)}{4s^3+8s^2+4s}\to 4.5;\quad \frac{s-1}{s^2-s-2}\to\infty$$

7. Determine the Laplace transforms of the following:

$$f(t) = 5e^{-2(t-3)}; \quad \{f(t) = 0, t < 3\} \quad \Rightarrow \quad F(s) = \frac{5e^{-3s}}{s+2}$$

$$m(t) = e^{-4(t-2)}\sin(3(t-2)); \quad \{m(t) = 0, t < 2\} \quad \Rightarrow \quad M(s) = \frac{3e^{-2s}}{(s+4)^2 + 9}$$

8. Remove the delay, use MATLAB to check your inverse Laplace, then add the delay back in to the time domain signal.