Modelling and control summaries

by Anthony Rossiter

Laplace 1 – definitions & common signals:

Laplace transforms are a tool used for linear systems analysis and design. As with all tools, they are designed to make some jobs easier so it is well worth the effort! Also, they are straightforward.

WORKING DEFINITION OF LAPLACE TRANSFORMS The integral exists if it remains bounded which it does in most practical problems. It is not the purpose here to be mathematically rigorous but simply to use the results.	$\boxed{L[f(t)] \equiv F(s) = \int_0^\infty f(t)e^{-st}dt}$ F(s) is the Laplace transform of signal *f(t)*
COMMON SIGNALS Within undergraduate engineering, you will likely only need to know the Laplace for a small set of signals. If you encounter something else, more study is needed.	$1, t, t^2, \quad e^{-at}, \quad e^{bt}, \quad \sin wt, \cos wt$ $t\sin wt, t\cos wt,$ $e^{-at}\cos wt, e^{-at}\sin wt$

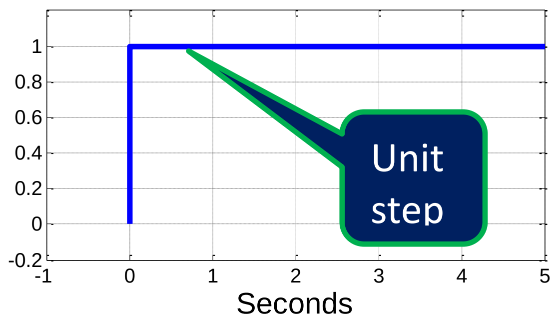
LAPLACE FROM FIRST PRINCIPLES

Students should be able to derive the Laplace transforms by using the integral definition above. Nevertheless, in practice, having done this, the results for commons signals can be memorised for efficiency. Note, only consider values in positive time, that is t>0!

The most common signal is a step or equivalently a constant. Using a unit step for convenience.

$$L[1] = \int_0^\infty 1.e^{-st}dt = \left[\frac{-1}{s}e^{-st}\right]_0^\infty$$

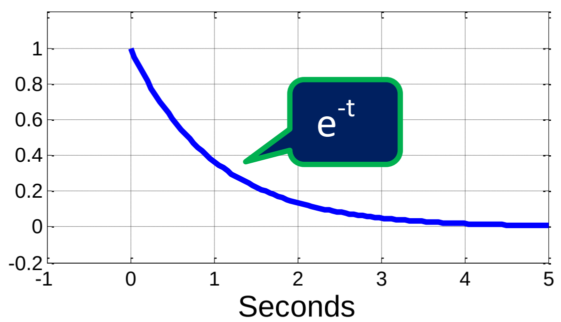
$$L[1] = \frac{-1}{s}[0-1] = \frac{1}{s}$$



Another common signal is an exponential, say $f(t)=e^{-at}$.

$$L[e^{-at}] = \int_0^\infty e^{-at}.e^{-st}dt = \left[\frac{-1}{(s+a)}e^{-(s+a)t}\right]_0^\infty$$

$$L[e^{-at}] = \frac{-1}{(s+a)}[0-1] = \frac{1}{s+a}$$



TUTORIAL QUESTIONS Using first principles, derive the following Laplace transforms (given k is a constant).	$L[3]=\frac{3}{s}$; $L[-2.4]=\frac{-2.4}{s}$; $L[k]=\frac{k}{s}$ $L[2t]=\frac{2}{s^2}$; $L[-kt^2]=\frac{-2k}{s^3}$ $L[e^{-0.1t}]=\frac{1}{s+0.1}$; $L[ke^{4.2t}]=\frac{k}{s-4.2}$

SUPERPOSITION Using the original definition the following result should be obvious.	Given $L[f_1(t)]=F_1(s)$; $L[f_2(t)]=F_2(s)$ Then $L[a_1 f_1(t)+a_2 f_2(t)] = a_1 L[f_1(t)]+a_2 L[f_2(t)]$ $L[a_1 f_1(t)+a_2 f_2(t)] = a_1 F_1(s)+a_2 F_2(s)$

Use this result to demonstrate the following are true:

$$L[3+e^{-t}]=\frac{4s+3}{s(s+1)}; \quad L[-k+2e^{bt}]=\frac{(2-k)s+bk}{s(s-b)};$$

$$L[kt+0.1e^{-at}]=\frac{0.1s^2+k(s+a)}{s^2(s+a)}; \quad L[-kt^2]=\frac{-2k}{s^3}$$

$$L[6e^{-4t}+5e^{-2t}]=\frac{11s+32}{s^2+6s+8};$$

SUMMARY: The most important results you will use are the following.

1.

$$L\left[Ae^{-at}\right]=\frac{A}{s+a}; \quad L\left[Ae^{bt}\right]=\frac{A}{s-b}$$

$$L[A]=\frac{A}{s}; \quad L[At^n]=\frac{An!}{s^{n+1}}$$

2. Superposition can be used as this follows directly from the integral definition.