

# Modelling and control summaries



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## Laplace 4 – interim summary and tutorial

### WORKING DEFINITION OF LAPLACE TRANSFORMS

F(s) is the Laplace transform of signal f(t)

$$L[f(t)] \equiv F(s) = \int_0^{\infty} f(t) e^{-st} dt$$

Although students should be able to derive key results because that helps with understanding and correct usage, in practice Laplace is done via look-up tables. That is students take standard results from tables and use these to help solve any problem in question. A summary of key results is given here – students should ensure they can derive all of these from first principles.

### Laplace of common signals (assume zero in negative time)

f(t)	F(s)		f(t)	F(s)
1	$\frac{1}{s}$		$\sin wt$	$\frac{w}{s^2 + w^2}$
t	$\frac{1}{s^2}$		$\cos wt$	$\frac{s}{s^2 + w^2}$
t <sup>k</sup>	$\frac{k!}{s^{k+1}}$		$e^{-at} \sin wt$	$\frac{w}{(s+a)^2 + w^2}$
e <sup>-at</sup>	$\frac{1}{s+a}$		$e^{-at} \cos wt$	$\frac{s+a}{(s+a)^2 + w^2}$
te <sup>-at</sup>	$\frac{1}{(s+a)^2}$		Unit Impulse	1

### Laplace of derivatives

$$L[x(t)] = X(s);$$

$$L\left[\frac{dx}{dt}\right] = sX(s) - x(0)$$

$$L\left[\frac{d^2x}{dt^2}\right] = s^2X(s) - sx(0) - \frac{dx}{dt}(0)$$

$$L\left[\frac{d^3x}{dt^3}\right] = s^3X(s) - s^2x(0) - s\frac{dx}{dt}(0) - \frac{d^2x}{dt^2}(0)$$

## Standard rules, shortcuts and properties

Superposition	$L[af(t) + bg(t)] = aL[f(t)] + bL[g(t)]$
Shift property	$L[e^{-at} f(t)] = F(s + a); \quad F(s) = L[f(t)]$
Integrals	$L\left[\int_0^t f(t) dt\right] = \frac{1}{s} F(s)$
Delays	$L[f(t - T)] = e^{-sT} L[f(t)] = e^{-sT} F(s)$
Scaling by t	$L[tf(t)] = -\frac{d}{ds} (F(s))$

### TUTORIAL QUESTIONS – verify the following

$$L[-4t] = \frac{-4}{s^2}; \quad L[e^{-3t} - e^{-4t}] = \frac{1}{s^2 + 7s + 12}; \quad L[3e^{2t} \sin 3t] = \frac{9}{s^2 - 4s + 13};$$

$$L[1 - 4e^{-3t} + 3e^{-4t}] = \frac{12}{s(s^2 + 7s + 12)}; \quad L[0.5e^{-2t} + 0.5e^{-6t}] = \frac{s + 4}{s^2 + 8s + 12};$$

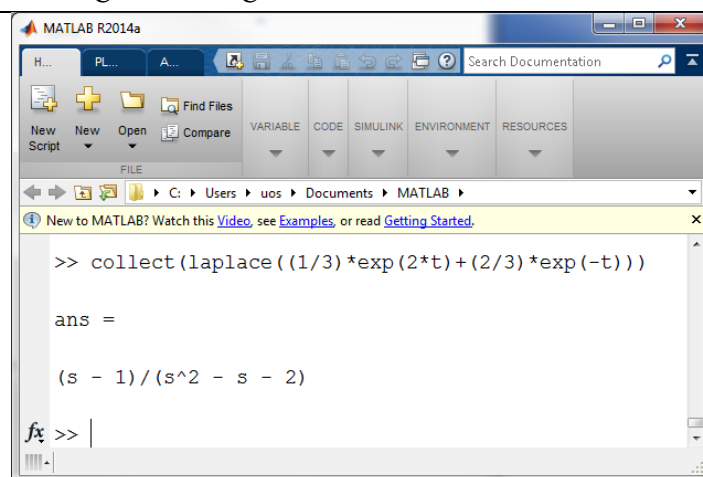
$$L[t - 0.5 + 0.5e^{-2t}] = \frac{8}{4s^3 + 8s^2}; \quad L[6 - e^{-t}(6 + 3t)] = \frac{12(s + 2)}{4s^3 + 8s^2 + 4s};$$

$$L\left[\frac{1}{3}e^{2t} + \frac{2}{3}e^{-t}\right] = \frac{s - 1}{s^2 - s - 2}$$

$$L\left[\int te^{-5t} dt\right] = \frac{1}{s(s + 5)^2} \quad \left\{y(t) = e^{-0.4(t-2)}; \quad t \geq 2\right\} \Rightarrow L[y(t)] = e^{-2s} \frac{1}{s + 0.4}$$

$$\left\{R \frac{dq}{dt} + \frac{q}{C} = 0.4\right\} \Rightarrow Q(s) = \frac{0.4}{s(Rs + \frac{1}{C})} + \frac{Rq(0)}{(Rs + \frac{1}{C})}; \quad \left\{\frac{dy}{dt} + y = t\right\} \Rightarrow Y(s) = \frac{1}{s^2(s + 1)} + \frac{y(0)}{(s + 1)}$$

### EXAMPLE MATLAB COMMAND TO CHECK WORK



```

MATLAB R2014a
>> collect(laplace((1/3)*exp(2*t)+(2/3)*exp(-t)))

ans =

(s - 1)/(s^2 - s - 2)

fx >>
  
```