Modelling and control summaries

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Laplace 4 – interim summary and tutorial

WORKING DEFINITION OF LAPLACE TRANSFORMS F(s) is the Laplace transform of signal *f(t)*	$L[f(t)] \equiv F(s) = \int_0^{\infty} f(t)e^{-st}dt$

Although students should be able to derive key results because that helps with understanding and correct usage, in practice Laplace is done via look-up tables. That is students take standard results from tables and use these to help solve any problem in question. A summary of key results is given here – students should ensure they can derive all of these from first principles.

Laplace of common signals (assume zero in negative time)

f(t)	F(s)		f(t)	F(s)
1	$\frac{1}{s}$		$\sin wt$	$\frac{w}{s^2+w^2}$
t	$\frac{1}{s^2}$		$\cos wt$	$\frac{s}{s^2+w^2}$
t^k	$\frac{k!}{s^{k+1}}$		$e^{-at}\sin wt$	$\frac{w}{(s+a)^2+w^2}$
e^{-at}	$\frac{1}{s+a}$		$e^{-at}\cos wt$	$\frac{s+a}{(s+a)^2+w^2}$
te^{-at}	$\frac{1}{(s+a)^2}$		Unit Impulse	1

Laplace of derivatives

$$L[x(t)] = X(s);$$

$$L[\frac{dx}{dt}] = sX(s) - x(0)$$

$$L[\frac{d^2x}{dt^2}] = s^2X(s) - sx(0) - \frac{dx}{dt}(0)$$

$$L[\frac{d^3x}{dt^3}] = s^3X(s) - s^2x(0) - s\frac{dx}{dt}(0) - \frac{d^2x}{dt^2}(0)$$

Standard rules, shortcuts and properties

Superposition	$L[af(t)+bg(t)]=aL[f(t)]+bL[g(t)]$
Shift property	$L[e^{-at}f(t)]=F(s+a); \quad F(s)=L[f(t)]$
Integrals	$L[\int_0^t f(t)dt]=\frac{1}{s}F(s)$
Delays	$L[f(t-T)]=e^{-sT}L[f(t)]=e^{-sT}F(s)$
Scaling by t	$L[tf(t)]=-\frac{d}{ds}(F(s))$

TUTORIAL QUESTIONS – verify the following

$$L[-4t]=\frac{-4}{s^2}; \quad L[e^{-3t}-e^{-4t}]=\frac{1}{s^2+7s+12}; \quad L[3e^{2t}\sin 3t]=\frac{9}{s^2-4s+13};$$

$$L[1-4e^{-3t}+3e^{-4t}]=\frac{12}{s(s^2+7s+12)}; \quad L[0.5e^{-2t}+0.5e^{-6t}]=\frac{s+4}{s^2+8s+12};$$

$$L[t-0.5+0.5e^{-2t}]=\frac{8}{4s^3+8s^2}; \quad L[6-e^{-t}(6+3t)]=\frac{12(s+2)}{4s^3+8s^2+4s};$$

$$L[\frac{1}{3}e^{2t}+\frac{2}{3}e^{-t}]=\frac{s-1}{s^2-s-2}$$

$$L\left[\int te^{-5t}dt\right]=\frac{1}{s(s+5)^2} \qquad \left\{y(t)=e^{-0.4(t-2)}; \quad t\geq 2\right\} \Rightarrow L[y(t)]=e^{-2s}\frac{1}{s+0.4}$$

$$\left\{R\frac{dq}{dt}+\frac{q}{C}=0.4\right\} \Rightarrow Q(s)=\frac{0.4}{s(Rs+\frac{1}{C})}+\frac{Rq(0)}{(Rs+\frac{1}{C})}; \quad \left\{\frac{dy}{dt}+y=t\right\} \Rightarrow Y(s)=\frac{1}{s^2(s+1)}+\frac{y(0)}{(s+1)}$$

EXAMPLE MATLAB COMMAND TO CHECK WORK