The University Of Sheffield.

Differentiation 1 concepts

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Introduction

The first video introduces the concept of differentiation.

- What does it mean to differentiate?
- What is a derivative?
- What notation is used for differentiation?

For now, this series focuses on simple cases, that is functions with a single independent variable and single dependent variable, e.g.

$$y = f(x); \quad z = g(w); \quad p = v(u)$$

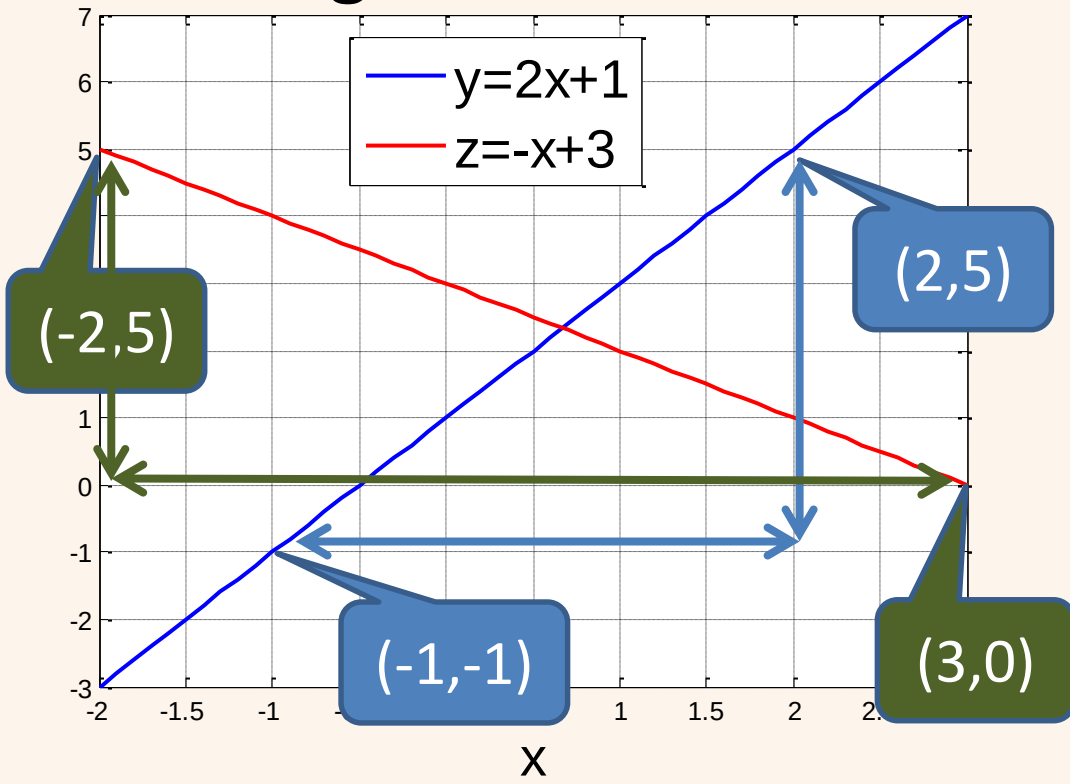
Gradients

It is assumed that by now students understand the term gradient, certainly when applied to a straight line.

$$y = 2x + 1;$$

$$z = -x + 3;$$

Gradient is change in dependent variable divided by change in independent variable.

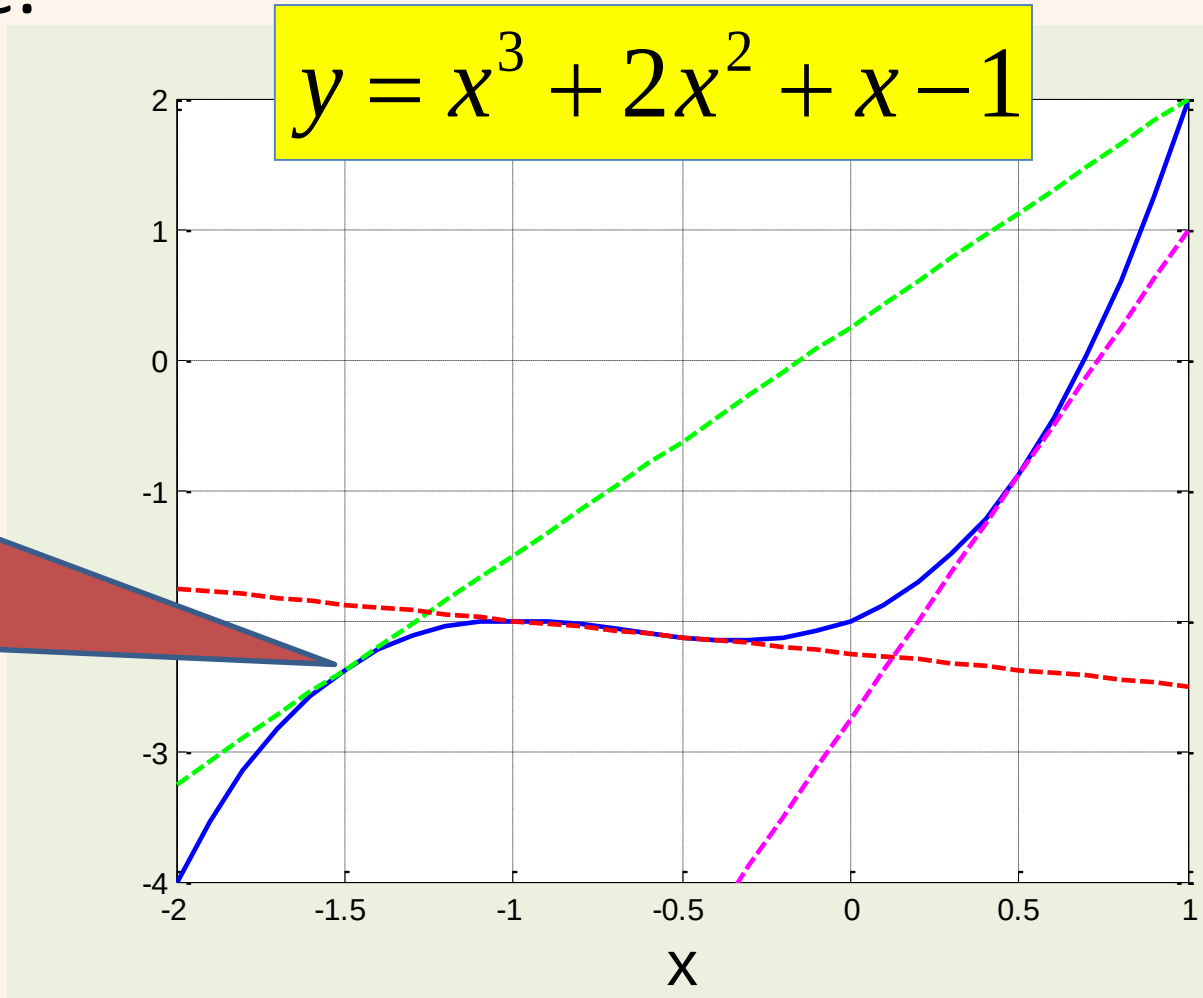


$$grad = \frac{y(x_2) - y(x_1)}{x_2 - x_1};$$

$$grad = \frac{(2x_2 + 1) - (2x_1 + 1)}{x_2 - x_1} = 2$$

Gradient of a general curve

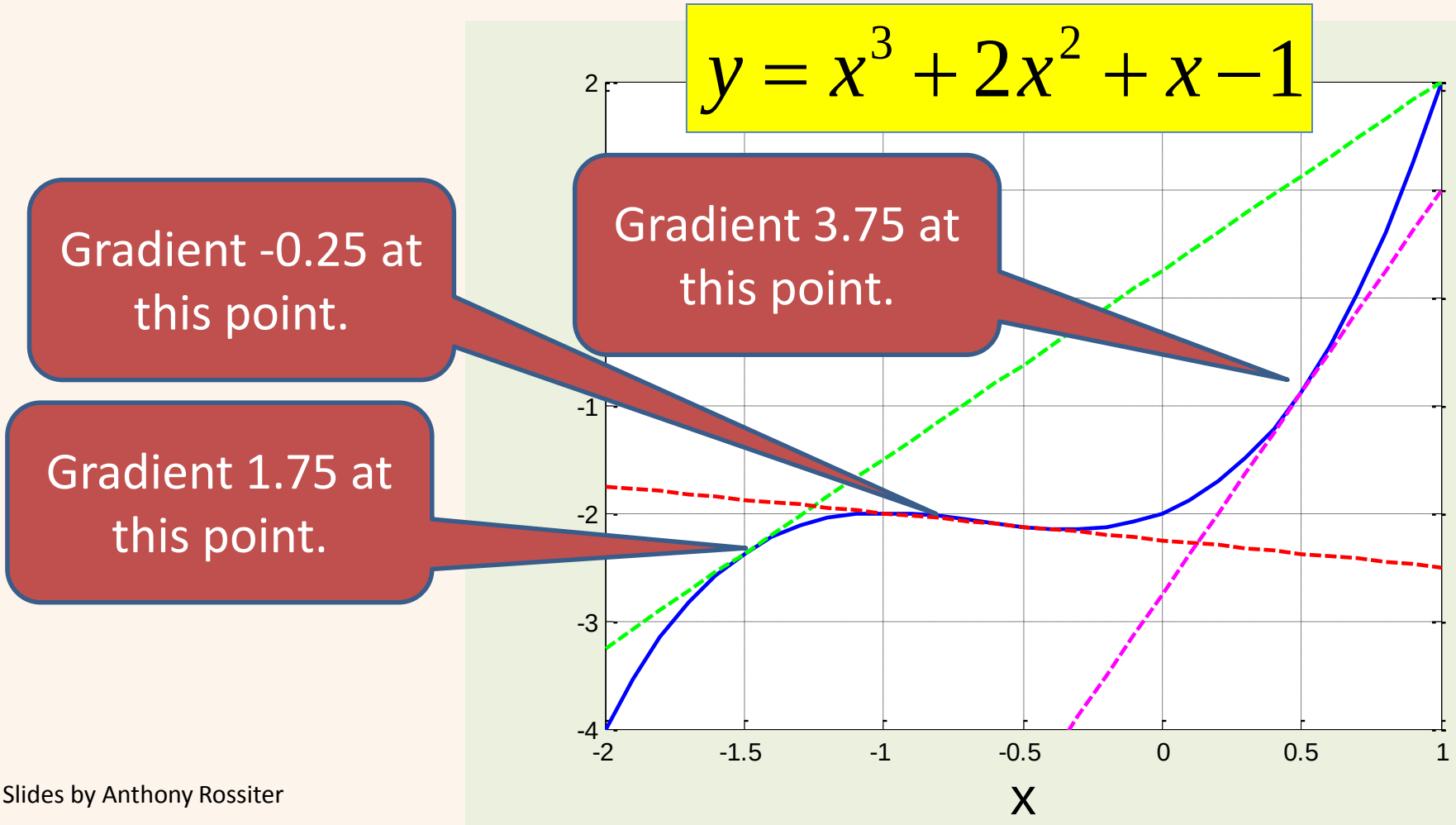
Functions which are not straight lines also have a gradient, but the gradient changes as you move along the curve.



Gradient is the instantaneous slope at any given point

What is differentiation?

Differentiation is a process which finds the gradient of a curve, precisely, at any point along the curve.



What is a derivative?

A derivative is the **result** of differentiation, in other words it is the function defining the gradient of a curve, precisely, at any point along the curve.

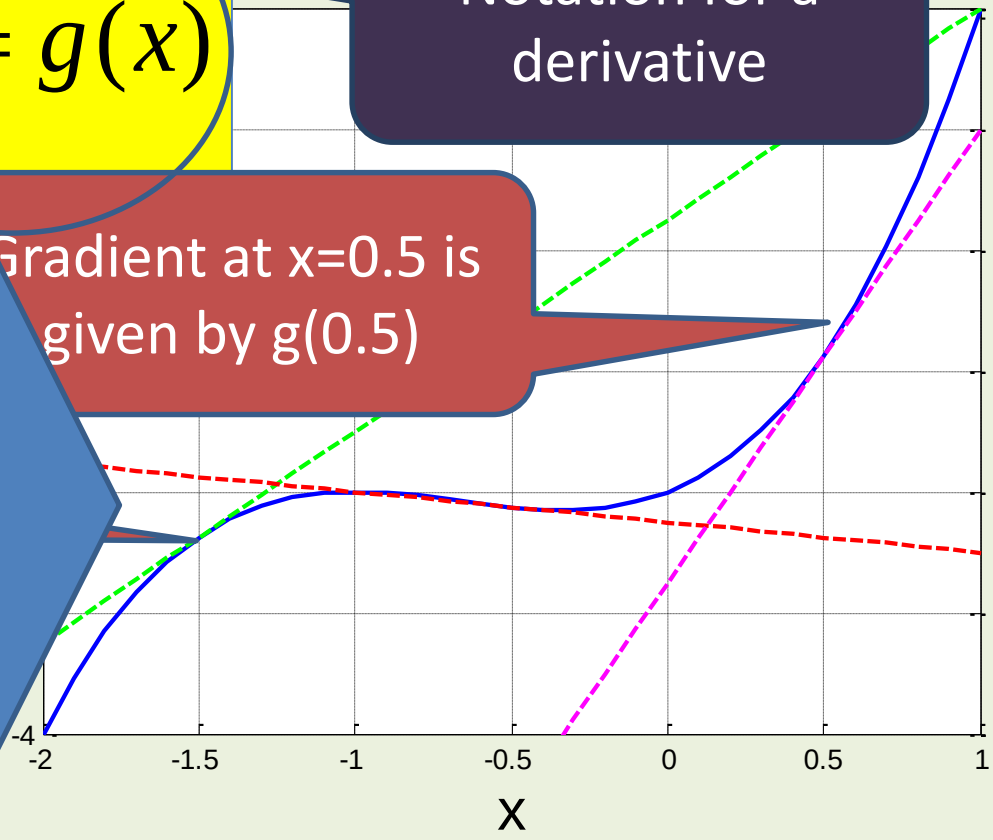
$$y = f(x) \quad \Rightarrow \quad \frac{dy}{dx} = g(x)$$

Notation for a derivative

Gradient at x=0.5 is given by g(0.5)

TALKING:
Students need to learn how to 'speak' the notation.

In this case dy/dx is spoken as 'd y d x'.



Interim summary

- Differentiation means to find the gradient; in general this involves some mathematical operations.
- A derivative is the result of differentiation, that is a function defining the gradient of a curve.
- The **notation** of derivative uses the letter ‘d’ and what looks like a fraction; **this is not a fraction and should not be treated as such!**

$$y = f(x) \quad \Rightarrow \quad \frac{dy}{dx} = derivative \equiv \frac{df}{dx}$$

Spoken as ‘d f d x’.

$$w = h(z) \quad \Rightarrow \quad \frac{dw}{dz} = derivative \equiv \frac{dh}{dz}$$

Spoken as ‘d h d z’.

Notation

A common mistake for students new to differentiation and derivatives is to assume that one can split up the terms dy and dx. YOU CANNOT DO THIS!

$\frac{dy}{dx}$

Is interpreted as single variable or single function.

This is specific mathematical notation.

Anywhere you see terms like the following, it is most likely they represent derivatives. My suggestion is to avoid using the letter ‘d’ as a variable and thus avoid any possible confusion with derivative notation.

$\frac{dr}{dt}; \quad \frac{dw}{ds};$

$\frac{dV}{dh}; \quad \frac{dP}{dR};$

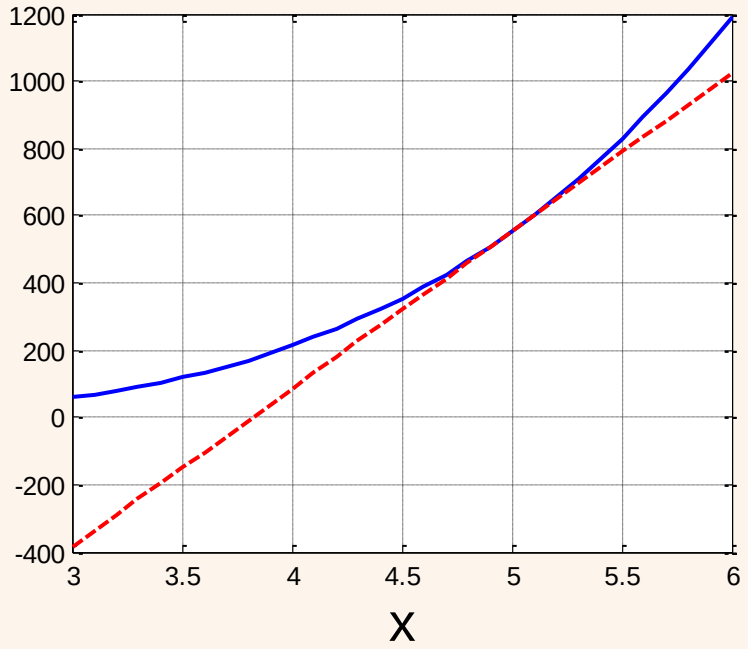
Examples

The following gives examples of the notation and process, but not detailed derivation which will follow in later videos.

Example 1

Consider the following function.

$$y = x^4 - 3x^2 + 5$$



One can differentiate this function to find the derivative as follows:

$$\frac{dy}{dx} = 4x^3 - 6x$$

So, for example, the gradient of the curve y(x) at x=5 is given as:

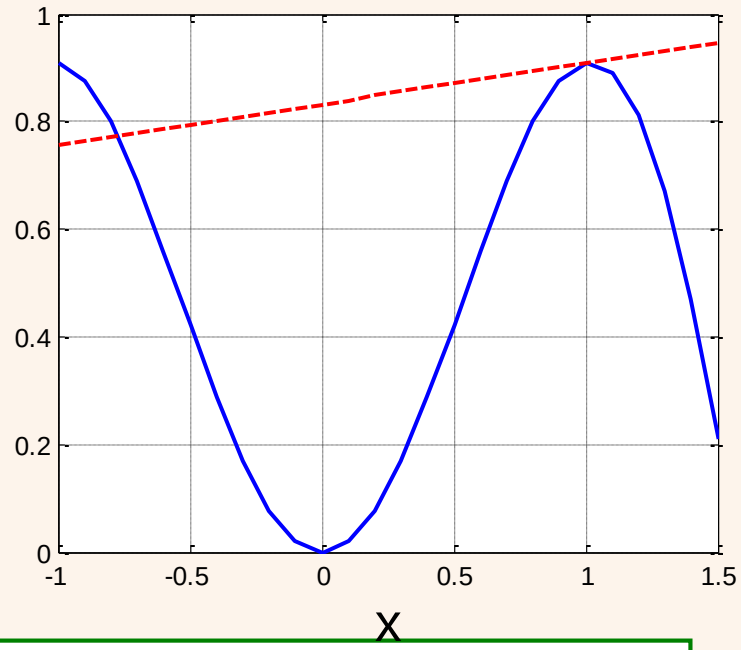
$$gradient\ at\ 5 = 4 \times 5^3 - 6 \times 5$$

$$= 470$$

Example 2

Consider the following function.

$$y = t\sin(2t)$$



One can differentiate this function to find the derivative as follows:

$$\frac{dy}{dt} = \sin(2t) + 2t\cos(2t)$$

So, for example, the gradient of the curve y(t) at t=1 is given as:

$$gradient\ at\ 1 = \sin(2) + 2\cos(2) = 0.077$$

Other notation

At times student will see the notation such as the following:

$$\frac{d}{dx}(f(x)); \quad \frac{d}{dz}(g(z)); \quad \frac{d}{dt}(a(t));$$

This means that the function is **to be differentiated, but the result is not yet known.**

This verbalised as ‘d d x of f’ or ‘ d d z of g’, etc.

Summary

- Differentiation means to find the gradient; in general this involves some mathematical operations.
- A derivative is the result of differentiation, that is a function defining the gradient of a curve.
- The **notation** of derivative uses the letter ‘d’ and **is not a fraction!**

$$y = f(x) \quad \Rightarrow \quad \frac{dy}{dx} = derivative \equiv \frac{df}{dx}$$

Spoken as ‘d f d x’.

$$\frac{d}{dx}(f)$$

Spoken as ‘d d x of f’.
The action of differentiation.

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