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Differentiation 3 superposition

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Introduction

- The previous video introduced differentiation from first principles.
- It is logical therefore to consider what consequences follow from this derivation.
- Superposition is a scenario where a function comprises the sum of other functions.
- How do we find the derivative of a function which is the sum of other functions?

Students can skip these explanations and go straight to resources 6 if they want to get straight into some computations.

Sum of functions

- The scenario we are looking at here is a function which is the sum of other functions.

$$y = f(x) = x^2 + 4\cos(2x)$$

$$z = g(w) = 3w^{-3} + 10\sin(w/2) + e^{-w}$$

$$h = k(t) = t^2 + 3t - 4 + 0.2e^{-0.1t}$$

How do we differentiate such functions?

Core result

The **derivative of a sum of functions** is **the sum of the derivatives**.

An example is given followed by a proof.

$$\frac{d}{dx}(x^2) = 2x$$

$$\frac{d}{dx}(\sin(4x)) = 4\cos(4x)$$

$$\frac{d}{dx}(x^2 + \sin(4x)) = 2x + 4\cos(4x)$$

That is, differentiate each term and then add the results.

REMINDER OF DEFINITION OF DERIVATIVE

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{y(x+\delta x) - y(x)}{(x+\delta x) - x}$$

$$\frac{dz}{dw} = \lim_{\delta w \to 0} \frac{z(w+\delta w) - z(w)}{(w+\delta w) - w}$$

Use derivative definition on a sum of functions

Let a function be given as follows: $y = f(x) + g(x)$

Use 1st principles definition of the derivative.

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{y(x+\delta x) - y(x)}{(x+\delta x) - x}$$

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{f(x+\delta x) + g(x+\delta x) - f(x) - g(x)}{\delta x}$$

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \left\{ \frac{f(x+\delta x) - f(x)}{\delta x} + \frac{g(x+\delta x) - g(x)}{\delta x} \right\}$$

$$\frac{dy}{dx} = \frac{df}{dx} + \frac{dg}{dx}$$

Extension to more functions is obvious.

Example 1

Find the derivative of the following.

$$y = f(x) = x^2 + 4\cos(2x)$$

$$\frac{d}{dx}(x^2) = 2x$$

$$\frac{d}{dx}(4\cos(2x)) = -8\sin(2x)$$

$$\frac{dy}{dx} = 2x - 8\sin(2x)$$

Example 2

Find the derivative of the following.

$$z = g(w) = 3w^{-3} + 10\sin(w/2) + e^{-w}$$

$$\frac{d}{dw}(3w^{-3}) = -9w^{-4}$$

$$\frac{d}{dw}(10\sin(w/2)) = 5\cos(w/2)$$

$$\frac{d}{dw}(e^{-w}) = -e^{-w}$$

$$\frac{dz}{dw} = -9w^{-4} + 5\cos(w/2) - e^{-w}$$

Summary

- This brief resource has shown that the derivative of a sum is the sum of the derivatives.
- In other words:

$$y = f + g + h + z$$

$$\frac{dy}{dx} = \frac{df}{dx} + \frac{dg}{dx} + \frac{dh}{dx} + \frac{dz}{dx}$$

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