



# Differentiation 6

## using a table of known results

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# Introduction

- The previous videos have given a definition and concise derivation of differentiation from first principles.
- The aim now is to give a number of examples.
- It is normal, once the derivations of key derivatives are understood, to give answers directly from a table of known results, that is, not to derive each case from first principles.

# Table of some common results

$$y = ax \Rightarrow \frac{dy}{dx} = a$$

$$y = ax^n \Rightarrow \frac{dy}{dx} = nax^{n-1}$$

$$y = \sin(bx) \Rightarrow \frac{dy}{dx} = b \cos(bx)$$

$$y = \cos(bx) \Rightarrow \frac{dy}{dx} = -b \sin(bx)$$

$$y = \tan(bx) \Rightarrow \frac{dy}{dx} = b \sec^2(bx)$$

$$y = e^{cx} \Rightarrow \frac{dy}{dx} = ce^{cx}$$

$$y = \sinh(bx) \Rightarrow \frac{dy}{dx} = b \cosh(bx)$$

$$y = \cosh(bx) \Rightarrow \frac{dy}{dx} = b \sinh(bx)$$

$$y = \log x \Rightarrow \frac{dy}{dx} = \frac{1}{x}$$

$$y = \frac{1}{\sin(bx)} \Rightarrow \frac{dy}{dx} = -b \frac{1}{\sin^2(bx)} \cot(bx)$$

NOT COMPLETE

# NUMERICAL EXAMPLES

# Example 1

$$y = 3x^6$$

Simply substitute into the formula in the table.

$$y = ax^n \Rightarrow \frac{dy}{dx} = nax^{n-1}$$

$$a = 3$$

$$n = 6$$

$$\frac{dy}{dx} = 6 \times 3 \times x^5$$

$$= 18x^5$$

# Example 2

$$y = 2x + 4x^5 - x^2$$

Simply substitute into the formula in the table doing each term one at a time (that is use the superposition result from resource 3).

$$y = ax^n \Rightarrow \frac{dy}{dx} = nax^{n-1}$$

$2x$	$a=2$ $n=1$	$\frac{d}{dx}(2x) = 2 + 1 \times x^0 = 2$
$4x^5$	$a=4$ $n=5$	$\frac{d}{dx}(4x^5) = 5 \times 4 \times x^4$
$-x^2$	$a=-1$ $n=2$	$\frac{d}{dx}(-x^2) = 2 + (-1) \times x$
		$\frac{dy}{dx} = 2 + 20x^4 - 2x$

# Example 3

$$y = 0.2e^{-5t}$$

Simply substitute into the formula in the table.

$$y = e^{ct} \Rightarrow \frac{dy}{dt} = ce^{ct}$$

$$c = -5$$

$$\frac{d}{dt} (e^{-5t}) = -5e^{-5t}$$

$$\frac{d}{dt} (0.2e^{-5t}) = -1e^{-5t}$$

# Example 4

$$y = \underline{2x^{-4}} + \underline{5e^{-0.2x}}$$

Simply substitute each term into the formula in the table.

$$y = ax^n \Rightarrow \frac{dy}{dx} = nax^{n-1}$$

$$y = e^{cx} \Rightarrow \frac{dy}{dx} = ce^{cx}$$

$$\frac{d}{dx} (2x^{-4}) = (-4) \times 2 \times x^{-5} = -8x^{-5}$$

$$\frac{d}{dx} (5e^{-0.2x}) = 5 \cdot (-0.2) e^{-0.2x}$$

$$\frac{dy}{dx} = -8x^{-5} - e^{-0.2x}$$



# Example 5

$$w = 0.4 \sin(3z) - 0.5 \cos(2z)$$

Simply substitute into the formula in the table.

$$w = \sin(bz) \Rightarrow \frac{dw}{dz} = b \cos(bz)$$

$$w = \cos(bz) \Rightarrow \frac{dw}{dz} = -b \sin(bz)$$

$$\frac{d}{dz} (0.4 \sin 3z) = 0.4 \times 3 \times \cos(3z)$$

$$\frac{d}{dz} (-0.5 \cos(2z)) = -0.5 \times (-2)$$

# Example 6

$$\log 3x = \log 3 + \log x$$

$$y = \log 3x + 4 \sinh 2x - x^{-5} + 2 \tan 5x$$

$$y = \log x \Rightarrow \frac{dy}{dx} = \frac{1}{x}$$

$$y = ax^n \Rightarrow \frac{dy}{dx} = nax^{n-1}$$

$$y = \sinh(bx) \Rightarrow \frac{dy}{dx} = b \cosh(bx)$$

$$y = \tan(bx) \Rightarrow \frac{dy}{dx} = b \sec^2(bx)$$

$$\frac{dy}{dx} = \frac{1}{x} + 4 \times 2 \times \cosh(2x) + 5x^{-6} + 2 \times 5 \times \sec^2 5x$$

# Summary

- This video has demonstrated the differentiation of commonplace functions using a lookup table.
- **In general it is advisable to have such a table to hand, even though in due course you are likely to remember many of the results.**



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