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Differentiation 6
using a table of known results

J A Rossiter

Introduction

- The previous videos have given a definition and concise derivation of differentiation from first principles.
- The aim now is to give a number of examples.
- It is normal, once the derivations of key derivatives are understood, to give answers directly from a table of known results, that is, not to derive each case from first principles.

Table of some common results

$$y = ax \Rightarrow \frac{dy}{dx} = a$$

$$y = ax^n \Rightarrow \frac{dy}{dx} = nax^{n-1}$$

$$y = \sin(bx) \Rightarrow \frac{dy}{dx} = b\cos(bx)$$

$$y = \cos(bx) \Rightarrow \frac{dy}{dx} = -b\sin(bx)$$

$$y = \tan(bx) \Rightarrow \frac{dy}{dx} = b\sec^2(bx)$$

$$y = e^{cx} \Rightarrow \frac{dy}{dx} = ce^{cx}$$

$$y = \sinh(bx) \Rightarrow \frac{dy}{dx} = b\cosh(bx)$$

$$y = \cosh(bx) \Rightarrow \frac{dy}{dx} = b\sinh(bx)$$

$$y = \log x \Rightarrow \frac{dy}{dx} = \frac{1}{x}$$

$$y = \frac{1}{\sin(bx)} \Rightarrow \frac{dy}{dx} = -b\frac{1}{\sin(bx)}\cot(x)$$

NOT COMPLETE

NUMERICAL EXAMPLES

Example 1 $y = 3x^6$

Simply substitute into the formula in the table.

$$y = ax^n \Rightarrow \frac{dy}{dx} = nax^{n-1}$$

$a = 3$

$n = 6$

$$\frac{dy}{dx} = 6 \times 3 \times x^5$$

$$= 18x^5$$

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Example 2

$$y = 2x + 4x^5 - x^2$$

Simply substitute into the formula in the table doing each term one at a time (that is use the superposition result from resource 3).

$$y = ax^n \quad \Rightarrow \quad \frac{dy}{dx} = nax^{n-1}$$

$2x$: $a = 2$, $n = 1$; $\frac{d}{dx}(2x) = 2 \times 1 \times x^0 = 2$

$4x^5$: $a = 4$, $n = 5$; $\frac{d}{dx}(4x^5) = 5 \times 4 \times x^4$

$-x^2$: $a = -1$, $n = 2$; $\frac{d}{dx}(-x^2) = 2 \times (-1) \times x$

$$\frac{dy}{dx} = 2 + 20x^4 - 2x$$

Example 3

$$y = 0.2e^{-5t}$$

Simply substitute into the formula in the table.

$$y = e^{ct} \quad \Rightarrow \quad \frac{dy}{dt} = ce^{ct}$$

$c = -5$

$$\frac{d}{dt}\left(e^{-5t}\right) = -5e^{-5t}$$

$$\frac{d}{dt}\left(0.2e^{-5t}\right) = -1e^{-5t}$$

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Example 4

$$y = 2x^{-4} + 5e^{-0.2x}$$

Simply substitute each term into the formula in the table.

$$y = ax^n \quad \Rightarrow \quad \frac{dy}{dx} = nax^{n-1}$$

$$y = e^{cx} \quad \Rightarrow \quad \frac{dy}{dx} = ce^{cx}$$

$$\frac{d}{dx}\left(2x^{-4}\right) = (-4)\times 2\times x^{-5} = -8x^{-5}$$

$$\frac{d}{dx}\left(5e^{-0.2x}\right) = 5.(-0.2)e^{-0.2x}$$

$$\frac{dy}{dx} = -8x^{-5} - e^{-0.2x}$$

Example 5

$$w = 0.4\sin(3z) - 0.5\cos(2z)$$

Simply substitute into the formula in the table.

$$w = \sin(bz) \quad \Rightarrow \quad \frac{dw}{dz} = b\cos(bz)$$

$$w = \cos(bz) \quad \Rightarrow \quad \frac{dw}{dz} = -b\sin(bz)$$

$$\frac{d}{dz}\left(0.4\sin 3z\right) = 0.4 \times 3 \times \cos(3z)$$

$$\frac{d}{dz}\left(-0.5\cos(2z)\right) = -0.5 \times (-2)\left(\quad\right)$$

Example 6

$\log 3x = \log 3 + \log x$

$$y = \log 3x + 4\sinh 2x + x^{-5} + 2\tan 5x$$

$$y = \log x \quad \Rightarrow \quad \frac{dy}{dx} = \frac{1}{x}$$

$$y = ax^{n} \quad \Rightarrow \quad \frac{dy}{dx} = nax^{n-1}$$

$$y = \sinh(bx) \quad \Rightarrow \quad \frac{dy}{dx} = b\cosh(bx)$$

$$y = \tan(bx) \quad \Rightarrow \quad \frac{dy}{dx} = b\sec^{2}(bx)$$

$$\frac{dy}{dx} = \frac{1}{x} + 4 \times 2 \times \cosh(2x) + 5x^{-6} + 2 \times 5 \times \sec^{2} 5x$$

Summary

- This video has demonstrated the differentiation of commonplace functions using a lookup table.
- **In general it is advisable to have such a table to hand, even though in due course you are likely to remember many of the results.**

Anthony Rossiter
Department of Automatic Control and Systems Engineering
University of Sheffield
www.shef.ac.uk/acse