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Differentiation 7
examples using the product rule

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Introduction

- The previous videos have given a definition and concise derivation of differentiation from first principles.
- The aim now is to give a number of examples.
- It is normal, once the derivations are understood, to give answers directly from a table of known results, that is, not to derive each case from first principles.
- Here the focus is on the product rule:

$$y = u(x)v(x)$$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

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Table of common results

$$y = ax \quad \Rightarrow \quad \frac{dy}{dx} = a$$

$$y = ax^n \quad \Rightarrow \quad \frac{dy}{dx} = nax^{n-1}$$

$$y = \sin(bx) \quad \Rightarrow \quad \frac{dy}{dx} = b\cos(bx)$$

$$y = \cos(bx) \quad \Rightarrow \quad \frac{dy}{dx} = -b\sin(bx)$$

$$y = \tan(bx) \quad \Rightarrow \quad \frac{dy}{dx} = b\sec^2(bx)$$

$$y = e^{cx} \quad \Rightarrow \quad \frac{dy}{dx} = ce^{cx}$$

$$y = \sinh(bx) \quad \Rightarrow \quad \frac{dy}{dx} = b\cosh(bx)$$

$$y = \cosh(bx) \quad \Rightarrow \quad \frac{dy}{dx} = b\sinh(bx)$$

$$y = \log x \quad \Rightarrow \quad \frac{dy}{dx} = \frac{1}{x}$$

$$y = \frac{1}{\sin(bx)} \quad \Rightarrow \quad \frac{dy}{dx} = -b\frac{1}{\sin(bx)}\cot(x)$$

NUMERICAL EXAMPLES

KEY TECHNIQUES

1. Define all functions used in the product rule, with their associated derivatives, clearly.
2. Ensure the layout of the work is uncluttered and unambiguous. This will avoid many typos.
3. Use known results from a table wherever possible.

Example 1

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Find the derivative of:

$$y = f(x) = 2x^3\cos(4x) = u(x)v(x)$$

Define u,v and du/dx, dv/dx.

$$u = 2x^3; \quad \frac{du}{dx} =$$

$$v = \cos(4x); \quad \frac{dv}{dx} =$$

Substitute into product formulae.

$$\frac{dy}{dx} = \quad 6x^2\cos(4x) - 8x^3\sin(4x)$$

Example 2

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Find the derivative of:

$$y = f(x) = 2e^{-2x}\tan(3x) = u(x)v(x)$$

Define u,v and du/dx, dv/dx.

$$u = 2e^{-2x}; \quad \frac{du}{dx} =$$

$$v = \tan(3x); \quad \frac{dv}{dx} =$$

$$\frac{dy}{dx} = \quad -4\tan(3x)e^{-2x} + 6\sec^2(3x)e^{-2x}$$

Example 3

$$\frac{dp}{dt} = v\frac{du}{dt} + u\frac{dv}{dt}$$

Find the derivative of:

Define u,v and du/dt, dv/dt.

$$p = f(t) = -4t\log t^2$$

$$u = -4t; \quad \frac{du}{dt} =$$

$$v = \log t^2; \quad \frac{dv}{dt} =$$

$$\frac{dp}{dt} = -4\log t^2 + \frac{-8t}{t}$$

Example 4

Using the product rule twice!

Define u,v and du/dw, dv/dw carefully for each step.

$$h = g(w) = \underbrace{w^2 e^{3w}}_{u} \underbrace{\sin 2w}_{v}$$

$$\frac{dh}{dw} = v\frac{du}{dw} + u\frac{dv}{dw}$$

$$u = w^2 e^{3w}$$

$$\frac{du}{dw} = e^{3w}\frac{d}{dw}\left(w^2\right) + w^2\frac{d}{dw}(e^{3w})$$

$$\frac{dh}{dw} = \sin(2w)\left[2we^{3w} + 3w^2e^{3w}\right] + 2w^2e^{3w}\cos 2w$$

Summary

- This video has demonstrated the differentiation of commonplace functions using a lookup table in combination with the product rule.
- Viewers will see that the steps are largely mechanical, albeit tedious at times.
- Keep clear definitions of u(x),v(x) and their derivatives before substituting into the formulae.
- **In general it is advisable to have a lookup table to hand, even though in due course you are likely to remember many of the results.**

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