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Differentiation 8
examples using the quotient rule

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Introduction

- The previous videos have given a definition and concise derivation of differentiation from first principles.
- The aim now is to give a number of examples.
- Here the focus is on the quotient rule in combination with a table of results for simple functions.

$$y = \frac{u(x)}{v(x)}$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Table of common results

$$y = ax \quad \Rightarrow \quad \frac{dy}{dx} = a$$

$$y = ax^n \quad \Rightarrow \quad \frac{dy}{dx} = nax^{n-1}$$

$$y = \sin(bx) \quad \Rightarrow \quad \frac{dy}{dx} = b\cos(bx)$$

$$y = \cos(bx) \quad \Rightarrow \quad \frac{dy}{dx} = -b\sin(bx)$$

$$y = \tan(bx) \quad \Rightarrow \quad \frac{dy}{dx} = b\sec^2(bx)$$

$$y = e^{cx} \quad \Rightarrow \quad \frac{dy}{dx} = ce^{cx}$$

$$y = \sinh(bx) \quad \Rightarrow \quad \frac{dy}{dx} = b\cosh(bx)$$

$$y = \cosh(bx) \quad \Rightarrow \quad \frac{dy}{dx} = b\sinh(bx)$$

$$y = \log x \quad \Rightarrow \quad \frac{dy}{dx} = \frac{1}{x}$$

$$y = \frac{1}{\sin(bx)} \quad \Rightarrow \quad \frac{dy}{dx} = -b\frac{1}{\sin(bx)}\cot(x)$$

NUMERICAL EXAMPLES

KEY TECHNIQUES

1. Define all functions used in the quotient rule, with their associated derivatives, clearly.
2. Ensure the layout of the work is uncluttered and unambiguous. This will avoid many typos.
3. Use known results from a table wherever possible.

Example 1

Find the derivative of:

$$y = f(x) = \frac{2x^4 + x}{\cos(5x)} = \frac{u(x)}{v(x)}$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Define u,v and du/dx, dv/dx.

$$u = 2x^4 + x; \quad \frac{du}{dx} =$$

$$v = \cos(5x);$$

$$\frac{dv}{dx} =$$

$$\frac{dy}{dx} = \frac{\cos(5x)(8x^3+1) + 5(2x^4+x)\sin(5x)}{\cos^2(5x)}$$

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Example 2

$$\frac{dy}{dx} = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

Find the derivative of:

$$y = f(x) = \frac{3\sin(2x) + \tan(3x)}{e^{0.4x}} = \frac{u(x)}{v(x)}$$

Define u,v and du/dx, dv/dx.

$$u = 3\sin(2x) + \tan(3x); \quad \frac{du}{dx} =$$

$$v = e^{0.4x};$$

$$\frac{dv}{dx} =$$

$$\frac{dy}{dx} = \frac{e^{0.4x}(6\cos(2x) + 3\sec^2(3x)) - (3\sin(2x) + \tan(3x))0.4e^{0.4x}}{e^{0.8x}}$$

Example 3

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Find the derivative of:

$$y = f(x) = \frac{\log(2x^3)}{(x^5 + 4x + 1)} = \frac{u(x)}{v(x)}$$

$$u = (\log 2 + 3\log x); \quad \frac{du}{dx} = \frac{3}{x}$$

$$v = x^5 + 4x + 1;$$

$$\frac{dv}{dx} =$$

SPECIAL CASES:
INVERSE OF TRIGONOMETRIC FUNCTIONS
INVERSE POLYNOMIALS

Example 4

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Find the derivative of:

$$y = f(x) = \frac{1}{x^n} = \frac{u(x)}{v(x)}$$

$$u = 1; \quad \frac{du}{dx} = 0$$

$$v = x^n; \quad \frac{dv}{dx} = nx^{n-1}$$

For completeness

$$\frac{d}{dx}\left(\frac{1}{x}\right)=\frac{-1}{x^2}$$

$$\frac{d}{dx}\left(\frac{1}{x^2}\right)=\frac{-2}{x^3}$$

$$\frac{d}{dx}\left(\frac{1}{x^3}\right)=\frac{-3}{x^4}$$

$$\frac{d}{dx}\left(\frac{1}{x^4}\right)=\frac{-4}{x^5}$$

Example 5

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Find the derivative of:

$$y = f(x) = \frac{1}{\sin(x)} = \frac{u(x)}{v(x)}$$

$$u = 1; \quad \frac{du}{dx} = 0$$

$$v = \sin(x); \quad \frac{dv}{dx} = \cos(x)$$

$$\frac{dy}{dx} = \frac{0 \times \sin(x) - \cos(x)}{\sin^2(x)}$$

Example 6

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Find the derivative of:

$$y = f(x) = \frac{1}{\cos(x)} = \frac{u(x)}{v(x)}$$

$$u = 1; \quad \frac{du}{dx} = 0$$

$$v = \cos(x); \quad \frac{dv}{dx} = -\sin(x)$$

$$\frac{dy}{dx} = \frac{0 \times \cos(x) + \sin(x)}{\cos^2(x)}$$

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Example 7

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Find the derivative of:

$$y = f(x) = \frac{1}{\tan(x)} = \frac{u(x)}{v(x)}$$

$$u = 1; \quad \frac{du}{dx} = 0$$

$$v = \tan(x); \quad \frac{dv}{dx} = \sec^2(x) = \frac{1}{\cos^2(x)}$$

$$\frac{dy}{dx} = \frac{0 \times \tan(x) - \sec^2(x)}{\tan^2(x)}$$

Slides by Anthony Rossiter

Summary

- This video has demonstrated the differentiation of commonplace functions using a lookup table in combination with the quotient rule.
- Viewers will see that the steps are largely mechanical, albeit tedious at times.
- Keep clear definitions of u(x),v(x) and their derivatives before substituting into the formulae.
- **In general it is advisable to have a lookup table to hand, even though in due course you are likely to remember many of the results.**

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