



# Matrices 1: definition of a matrix

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<http://controleducation.group.shef.ac.uk/indexwebbook.html>

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# Introduction

- This video is very brief and has the purpose of introducing terminology for matrices.
- The key thing for viewers to note is that these are **DEFINITIONS**:
  - These definitions are to some extent arbitrary so should taken as given.
  - The convenience of these definitions will become apparent later.

# What is table?

Most viewers are accustomed to using tables to represent and store data.

This table has 3 rows of data and 5 columns.

Day	Number of apples	Number of workers	Apples discarded	Profit (£)
1	24	2	5	3.23
2	35	3	8	4.06
3	42	3	8	5.12

Row 1

Row 2

Row 3

Column 1

Column 3

# What is a matrix?

In simple terms a matrix is just a more compact representation of a table of numbers!

Day	Number of apples	Number of workers	Apples discarded	Profit (£)
1	24	2	5	3.23
2	35	3	8	4.06
3	42	3	8	5.12

$$A = \begin{bmatrix} 1 & 24 & 2 & 5 & 3.23 \\ 2 & 35 & 3 & 8 & 4.06 \\ 3 & 42 & 3 & 8 & 5.12 \end{bmatrix}$$

Same number of rows and columns.

# Remark

When storing tabular data in a matrix, the user needs to keep a record of what each row and column represents separately as this is no longer stated explicitly as in a table.

$$A = \begin{bmatrix} 1 & 24 & 2 & 5 & 3.23 \\ 2 & 35 & 3 & 8 & 4.06 \\ 3 & 42 & 3 & 8 & 5.12 \end{bmatrix}$$



# Matrices and MATLAB for plots

A student collects data from an experiment and stores the answer in a matrix *data\_exp1*.

- Column 1 corresponds to time.
- Column 2 to temperature
- Column 3 is voltage supplied.

It is now simple to code and plot this data – the corresponding code is shown next.

# MATLAB example

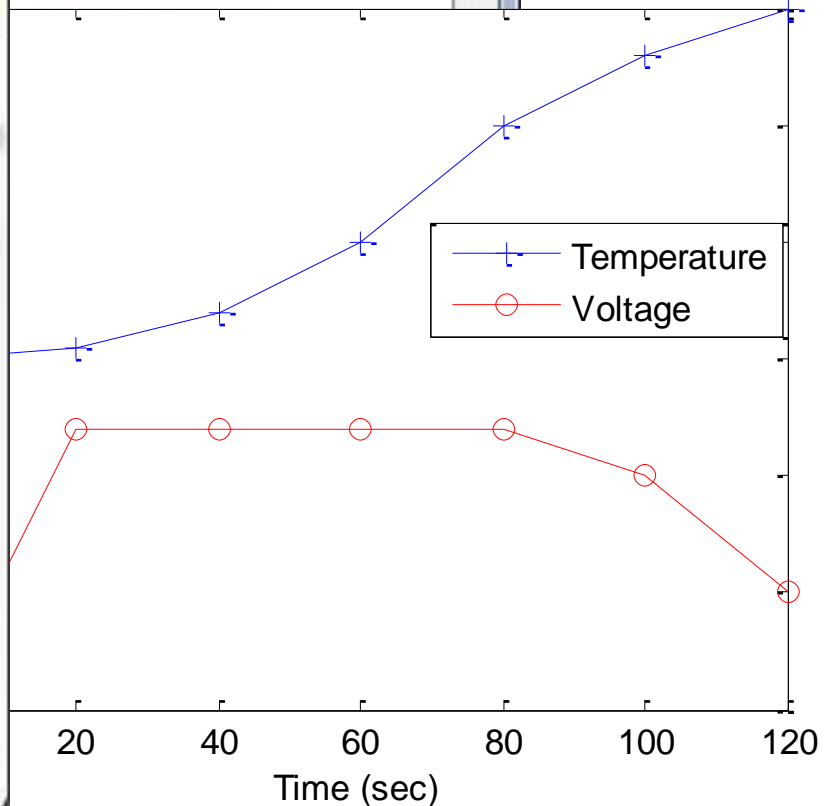
```
MATLAB 7.12.0 (R2011a)
File Edit Debug Parallel Desktop Window Help
Shortcuts How to Add What's New
New to MATLAB? Watch this Video, see Demos, or read Getting Started
>> data_exp1

data_exp1 =

    0    15.0000    0
  20.0000    15.5000   12.0000
  40.0000    17.0000   12.0000
  60.0000    20.0000   12.0000
  80.0000    25.0000   12.0000
 100.0000    28.0000   10.0000
 120.0000    30.0000    5.0000

fx >>
```

```
SEPTEMBER 3RD 2012
0]; %%% TIME
30]; %% tTEMPERATURE
%% VOLTAGE
```



# What use are matrices?

- Compact form for storing data (equivalent to a table but more flexible).
- Each column(or row) can relate to a different 'state': e.g. Time, displacement, velocity, temperature, etc.
- Convenient for computer code (and algebra) as can store and share large quantities of related data with a single variable.
- Easily extend to dynamic relations/modelling (see years 2/3 material).



# Matrix Dimensions

Matrices have 2 dimensions.

**Number of rows and number of columns.**

$$A = \begin{bmatrix} 1 & 24 & 2 & 5 & 3.23 \\ 2 & 35 & 3 & 8 & 4.06 \\ 3 & 42 & 3 & 8 & 5.12 \\ 6 & 23 & 4 & 9 & 5.26 \end{bmatrix}$$

4 rows and 5 columns.

$$B = \begin{bmatrix} 1 & 0 & 3 \\ 5 & 0 & 0 \\ -5 & 2 & -4 \end{bmatrix}$$

3 rows and 3 columns.

# Matrix dimensions

What are the dimensions of the following matrices?

Convention is to give the row dimension first.

$$A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 4 & 5 & 0 \\ 3 & -12 & -15 & 0 \\ 5 & 18 & 3 & 7 \\ 104 & 56 & 13 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 \\ 1 \\ -2 \\ 0 \end{bmatrix}$$

## TERMINOLOGY:

We say that A is 2 by 2, B is 4 by 4 and C is 4 by 1

# Matrix dimensions

What are the dimensions of the following matrices?

$$D = \begin{bmatrix} 8 & -3 & 2 \\ 6 & 5 & 0 \\ 2 & 4 & 0 \end{bmatrix}; \quad E = \begin{bmatrix} 3 & -3 & 2 & 4 \\ 4 & 5 & 1 & 5 \\ 2 & 4 & 0 & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 & 4 \\ -1 & 0 & 5 \end{bmatrix}; \quad B = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$$

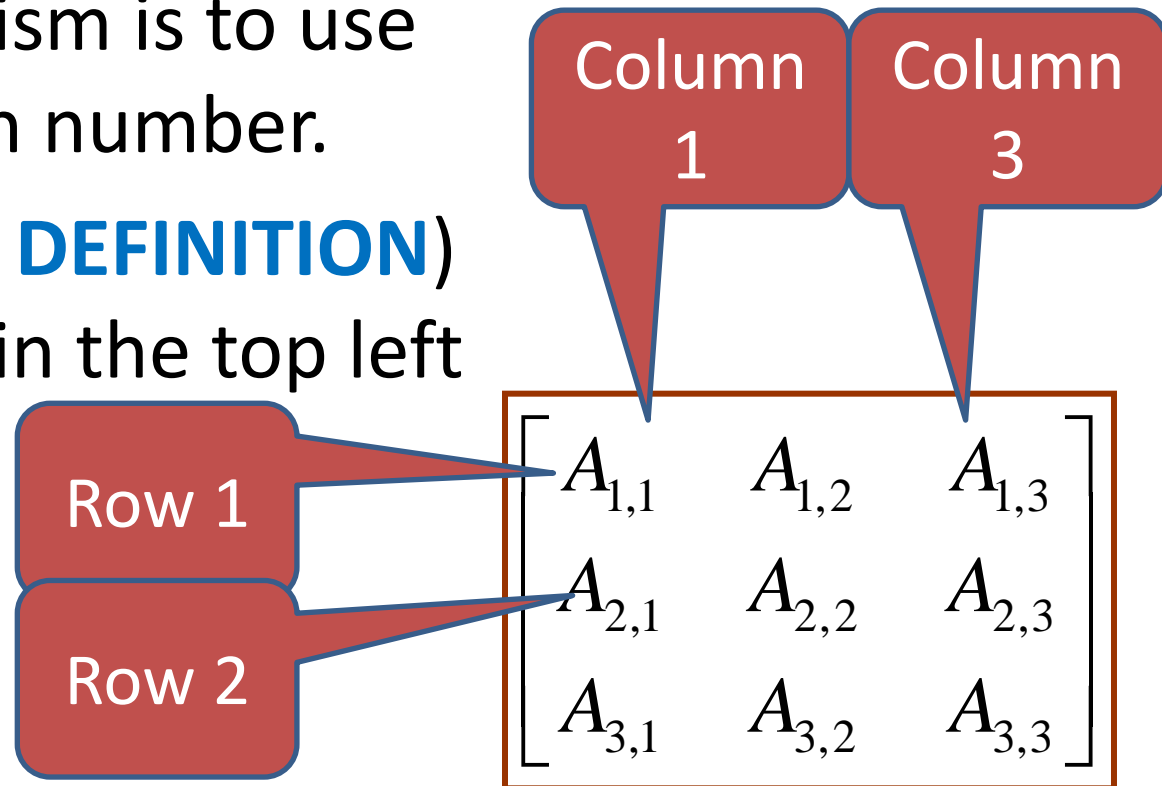
$$C = [21 \quad 5 \quad -4 \quad 0 \quad 4 \quad 0]$$

# Notation for matrix elements

We need a mechanism for referring to different elements in a matrix.

The logical mechanism is to use the row and column number.

The convention (**BY DEFINITION**) is to start counting in the top left hand corner.



# Notation for matrix elements

Elements are referred to by their row and column index, row first and column second.

Row 1 and  
column 2

$$\begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} \\ A_{2,1} & A_{2,2} & A_{2,3} \\ A_{3,1} & A_{3,2} & A_{3,3} \end{bmatrix}$$

Row 2 and  
column 3

# Matrix notation

Find the element  $A_{3,4}$  and the element  $C_{1,5}$

Remember the convention is to give the row dimension first.

$$A = \begin{bmatrix} -1 & 4 & 5 & 0 \\ 3 & -12 & -15 & 0 \\ 5 & 18 & 3 & 7 \\ 104 & 56 & 13 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 2 & 4 & 4 \\ 1 & 0 & 6 & 2 & -1 \end{bmatrix}$$

Which statement is false?

$$D = \begin{bmatrix} 8 & -3 & 2 \\ 6 & 5 & 0 \\ 2 & 4 & 0 \end{bmatrix}$$

1.  $D_{12} = -3$
2.  $D_{21} = 6$
3.  $D_{32} = 2$
4.  $D_{23} = 2$
5.  $D_{33} = 0$

What is the index of the component -4?

$$B = \begin{bmatrix} 2 & 3 & 4 & 6 \\ -4 & 5 & 2 & -5 \\ 7 & 8 & 6 & 8 \\ 54 & 23 & 1 & 2 \end{bmatrix}$$



# Shorthand notation for matrix elements

Viewers will often see that the comma is omitted where the meaning is obvious.

$$\begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} \\ A_{2,1} & A_{2,2} & A_{2,3} \\ A_{3,1} & A_{3,2} & A_{3,3} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

However, for dimensions 10 and greater, a comma is needed to be precise.

$$A_{236} = A_{2,36} \quad \text{or} \quad A_{23,6}$$

# Create a matrix with the following information

2<sup>nd</sup> row and 3<sup>rd</sup> column is 5:  $A_{25}=5$

4<sup>th</sup> row and 2<sup>nd</sup> column is 2:  $A_{42}=2$

1<sup>st</sup> row and 5<sup>th</sup> column is 6:  $A_{15}=6$

Remainder are 0.

# Summary

Introduced the following concepts.

1. Definition of a matrix
2. Dimensions of a matrix and how these are verbalised.
3. Notation for defining elements of a matrix.

$$A = \begin{bmatrix} 1 & 24 & 2 & 5 & 3.23 \\ 2 & 35 & 3 & 8 & 4.06 \\ 3 & 42 & 4 & 8 & 5.12 \end{bmatrix}$$

A is 3 by 5  
because it has  
3 rows and 5 columns.

Element  $A_{2,4}$  as on 2<sup>nd</sup> row and 4<sup>th</sup> column