Matrices 3: addition and subtraction

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http://controleducation.group.shef.ac.uk/indexwebbook.html

http://www.shef.ac.uk/acse

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Introduction

- The previous videos introduced definitions of matrices, basic notation and special cases such as square matrices, symmetry and vectors.
- This video looks at the concepts of addition and subtraction.

VIEWERS should note that these results are **BY DEFINITION** – they cannot be proved or derived.

Requirement

Matrices can only be added or subtracted if they have the same dimensions.

Adding matrices of different dimensions has no logical meaning.

Adding a constant to a matrix is also poorly defined as

- a constant is dimension 1 by 1 and
- a matrix is dimension r by n.

If you want to add the same constant to every element, state this explicitly.

Addition and subtraction of matrices

Add (or subtract) components in same position, that is with the same {row, column} index.

$$C = A + B \quad \Rightarrow \quad C_{i,j} = A_{i,j} + B_{i,j}$$

Matrices must be **same dimensions** or you cannot add them!

$$A = \begin{bmatrix} 2 & 3 \\ -3 & 4 \end{bmatrix}; \qquad B = \begin{bmatrix} 1 & 2 \\ 7 & -6 \end{bmatrix}; \quad A + B = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}$$

Do following matrix computations

$$A = \begin{bmatrix} 2 & 3 \\ -3 & 4 \end{bmatrix}; \quad B = \begin{bmatrix} 4 & 2 \\ 7 & -6 \end{bmatrix}; \quad A - B = \begin{bmatrix} & \\ & \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 \\ -6 & 4 \\ 3 & 4 \end{bmatrix}; \quad B = \begin{bmatrix} -2 & 0 \\ 12 & 6 \\ 8 & 7 \end{bmatrix}; \quad A + B = \begin{bmatrix} & \\ & \\ & \end{bmatrix}$$

$$G = \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}; \quad A = \begin{bmatrix} 2 & -2 & -2 \\ 0 & 6 & 8 \end{bmatrix};$$

$$C = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}; \quad D = \begin{bmatrix} 2 & 0 & 2 \\ 2 & -2 & -1 \end{bmatrix}$$

$$F = \begin{bmatrix} 3 & 4 \\ -1 & 9 \end{bmatrix}; \quad E = \begin{bmatrix} 0 & -2 & -4 \\ -2 & 8 & 9 \end{bmatrix}$$

Are these correct?

1. **G+C=F**
2. **A+D=E**

Do following matrix computations

$$A = \begin{bmatrix} 2 & 3 \\ -3 & 4 \end{bmatrix}; \qquad B = \begin{bmatrix} 4 & 1 & 0 \\ 7 & 5 & -1 \end{bmatrix}; \quad A^T - B = \begin{bmatrix} & \\ & \end{bmatrix}$$

$$A = \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}; \qquad B = \begin{bmatrix} 2 & 4 & 8 \end{bmatrix}; \quad A + B^T =$$

Multiplication by a constant

It is useful to define what multiplication by a constant means. For example:

$$C = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 3 & 6 \\ 4 & 2 \\ 4 & -1 \end{bmatrix} \Rightarrow C + C + C = \begin{bmatrix} 0 & 3 \\ 0 & 0 \\ 9 & 18 \\ 12 & 6 \\ 12 & -3 \end{bmatrix}$$

Here $C+C+C=3C$

Multiplication by a constant

In general, multiplication by constant is taken to be applied to every coefficient individually.

HENCE (**by definition**):

$$C = \lambda A \quad \Rightarrow \quad C_{i,j} = \lambda A_{i,j}$$

$$B = \begin{bmatrix} 1 & 5 & 3 \\ 5 & 0 & -2 \\ 3 & -2 & -4 \end{bmatrix} \quad \Rightarrow \quad 4.3B = \begin{bmatrix} 1\times 4.3 & 5\times 4.3 & 3\times 4.3 \\ 5\times 4.3 & 0 & -2\times 4.3 \\ 3\times 4.3 & -2\times 4.3 & -4\times 4.3 \end{bmatrix}$$

Determine C=A-4D+3E

$$A=\begin{bmatrix}2 & -2 & -2\\ 0 & 6 & 8\end{bmatrix};\quad D=\begin{bmatrix}2 & 0 & 2\\ 2 & -2 & -1\end{bmatrix};\quad E=\begin{bmatrix}0 & -2 & -4\\ -2 & 8 & 9\end{bmatrix}$$

HENCE:

$$C_{i,j}=A_{i,j}-4D_{i,j}+3E_{i,j}$$

Prove the following

A matrix W is symmetric. Show that

$$W-W^T=0$$

Also do some numerical examples to verify this.

Symmetry is defined as

$$\{W_{i,j} = W_{j,i}, \forall i, j\}$$

$$D = W - W^T \quad \Rightarrow \quad D_{i,j} = W_{i,j} - W_{j,i}, \quad \forall i, j$$

Summary

- Defined matrix addition and subtraction.
- Defined matrix multiplication by a constant.
- Given several numerical examples.