The University Of Sheffield.

Matrices 5: multiplication

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http://controleducation.group.shef.ac.uk/indexwebbook.html

http://www.shef.ac.uk/acse

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Introduction

- The previous videos introduced basic matrix notation and operations.
- This video looks at the concepts of multiplication of matrices.
- There is no such thing as matrix division.

VIEWERS should note that these results are **BY DEFINITION** – they cannot be proved or derived.

Basic rule for multiplication
(BY DEFINITION – not to understand)

Index {i,j} of result is determined from row {i} of left hand matrix and column {j} of right hand matrix.

Multiplying a row on left by a column on the right is a 'dot-product' type of operation, e.g.

$$\begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} e \\ f \\ g \\ h \end{bmatrix} = ae + bf + cg + dh$$

CAVIAT:
Number of columns of left hand matrix must match number of row of right hand matrix!
If not, multiplication cannot be performed!

What is the result of the following ?

$$\begin{bmatrix} 3 & 3 & 5 \end{bmatrix} \begin{bmatrix} -2 \\ 6 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 1 & -2 & 7 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 0 \\ 2 \end{bmatrix}$$

What is the result of the following ?

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

KEY OBSERVATION

A row vector can only be multiplied onto a column vector if the two vectors are the same length.

Number of columns of left hand vector must match the number of rows of the right hand vector.

If vectors have different dimensions, multiplication is not defined.

Generic formulae

For a row vector A and column vector B of the same dimension, then.

$$AB = A_1B_1 + A_2B_2 + \cdots A_nB_n$$

For more general matrices, then the following formulae is used.

$$C = AB \quad \Rightarrow \quad C_{i,j} = A_{i,1}B_{1,j} + A_{i,2}B_{2,j} + \cdots A_{i,n}B_{n,j}$$

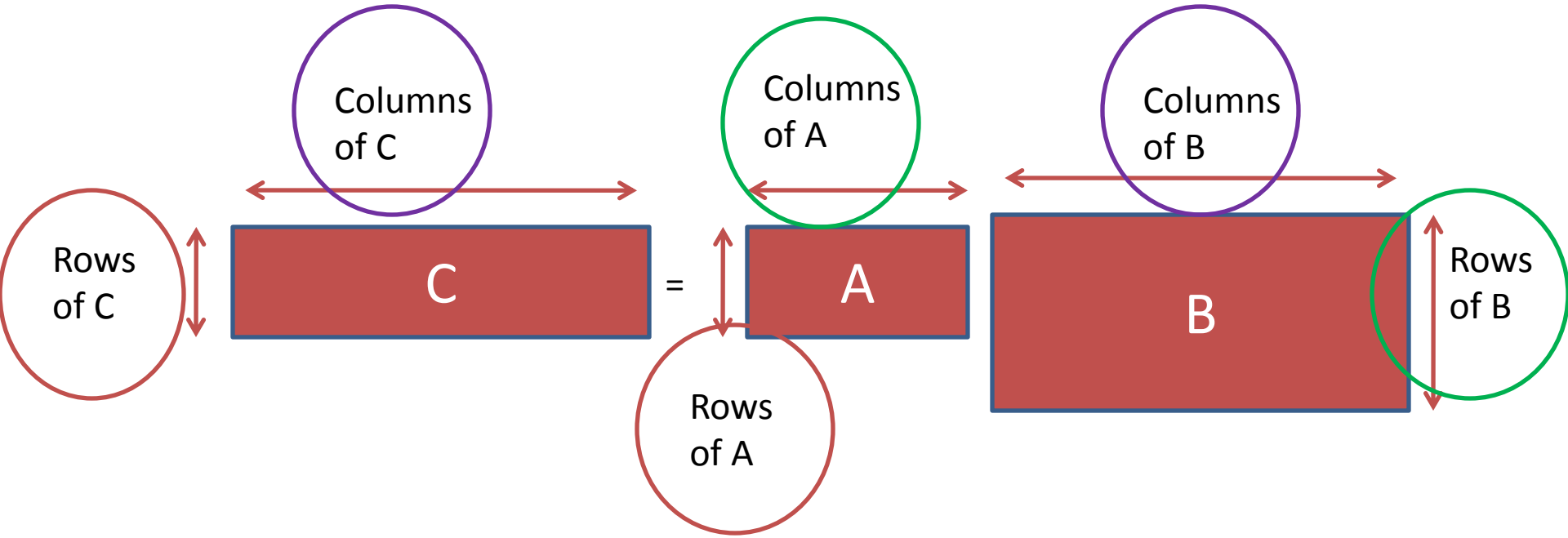
{i,j} element of the result uses the ith row of the left hand matrix and the jth column of the right hand matrix.

Rules for matrix multiplication

Assume we wish to do C=A*B.

A must have same number of columns as B has rows.

Result has row dimension of A and column dimension of B.



Basic rule for multiplication

Index {i,j} or result is determined from row {i} of left hand matrix and column {j} of right hand matrix. Multiplying a row on left by a column on the right is a 'dot-product' type of operation, e.g.

$$\begin{bmatrix} a & b & c & d \\ k & n & m & p \end{bmatrix} \begin{bmatrix} e \\ f \\ g \\ h \end{bmatrix} = \begin{bmatrix} ae+bf+cg+dh \\ ke+nf+mg+ph \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \begin{bmatrix} g & h & i & j \\ k & n & m & p \end{bmatrix} = \begin{bmatrix} ag+bk & ah+bn & ai+bm & aj+bp \\ cg+dk & ch+dn & ci+dm & cj+dp \\ eg+fk & eh+fn & ei+fm & ej+fp \end{bmatrix}$$

Student problem

Solve the following.

$$\begin{bmatrix} 2 & 5 & 6 \\ 4 & 6 & 8 \end{bmatrix} \begin{bmatrix} -1 & 7 \\ -4 & 5 \\ 6 & 2 \end{bmatrix} =$$

Student problem

Solve the following.

$$\begin{bmatrix} 2 & 5 & 6 \\ 4 & 6 & 8 \end{bmatrix} \begin{bmatrix} -1 & 7 & 9 & 1 \\ -4 & 5 & 10 & 1 \\ 6 & 2 & 12 & 2 \end{bmatrix} =$$

KEY OBSERVATION

Two matrices can only be multiplied if the column dimension of the left matrix matches the row dimension of the right matrix.

In general

$$AB \neq BA$$

The property of AB=BA is called commutativity and does not hold for matrices in general.

In fact, it is possible that AB exists but BA does not! Give an example of this.

Student problem 2 – compute C,D

$$A=\begin{bmatrix}2 & 5 & 6 & 4\\ 4 & 6 & 8 & 6\end{bmatrix};\quad B=\begin{bmatrix}-1 & 7\\ -4 & 5\\ 6 & 2\end{bmatrix};\quad C=AB;\quad D=BA$$

Summary

- Defined matrix multiplication.
- Noted that multiplication is only defined if the column dimension of the left hand matrix equals the row dimension of the right hand matrix.
- The dimensions of the results are taken from row dimension of the left hand matrix and column dimension of the right hand matrix.
- The existence of AB does not imply the existence of BA.