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# Matrices 7: uses of matrix multiplication

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**http://controleducation.group.shef.ac.uk/indexwebbook.html**

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# Introduction

- The previous videos covered multiplication of matrices and key properties.
- Here we look at some uses of multiplication for problem solving.

# Generic formulae for matrix multiplication

The following formulae is used.

$$C = AB \quad \Rightarrow \quad C_{i,j} = A_{i,1}B_{1,j} + A_{i,2}B_{2,j} + \cdots A_{i,n}B_{n,j}$$

{i,j} element of the result uses the ith row of the left hand matrix and the jth column of the right hand matrix.

Result has row dimension of A and column dimension of B.

# SIMULTANEOUS EQUATIONS

Matrix/vector algebra is a compact and efficient method for handling linear simultaneous equations and most effective techniques deployed assume matrix representation of this problem.

# Using Matrices for simultaneous equations

Consider a single linear equation and write in MATRIX/VECTOR format, e.g.

$$3x + 2y = 4 \quad \Rightarrow$$

$$\begin{bmatrix} 3x + 2y \\ -x + 5y \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

# Put following into matrix-vector format

$$\begin{Bmatrix} 5x + 6y = -1 \\ x - 3y = 2 \end{Bmatrix}$$

$$\begin{Bmatrix} x - z + 2y = 3 \\ 3z - 6y = -4 \\ 20x + 13y - 4z = 10 \end{Bmatrix}$$

# Put following into matrix-vector format

$$\left\{\begin{array}{c} a+b-c=12 \\ b+2c-3d=4 \\ a+14c+d=2 \\ b+12a+14d=9 \end{array}\right\}$$

# Solution of simultaneous equations

This is covered more carefully after the video on matrix inverse.

Here suffice to note that, if a matrix A is square and has an inverse, then:

$$\begin{bmatrix} 3 & 2 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix} \quad \Leftrightarrow \quad AX = b$$

Mathematically compact and convenient!

# Use of matrices with data manipulation

Given a model takes a form which is linear in some parameters a, b, c, d.

$$Y = ax + bz + cw^2 + d$$

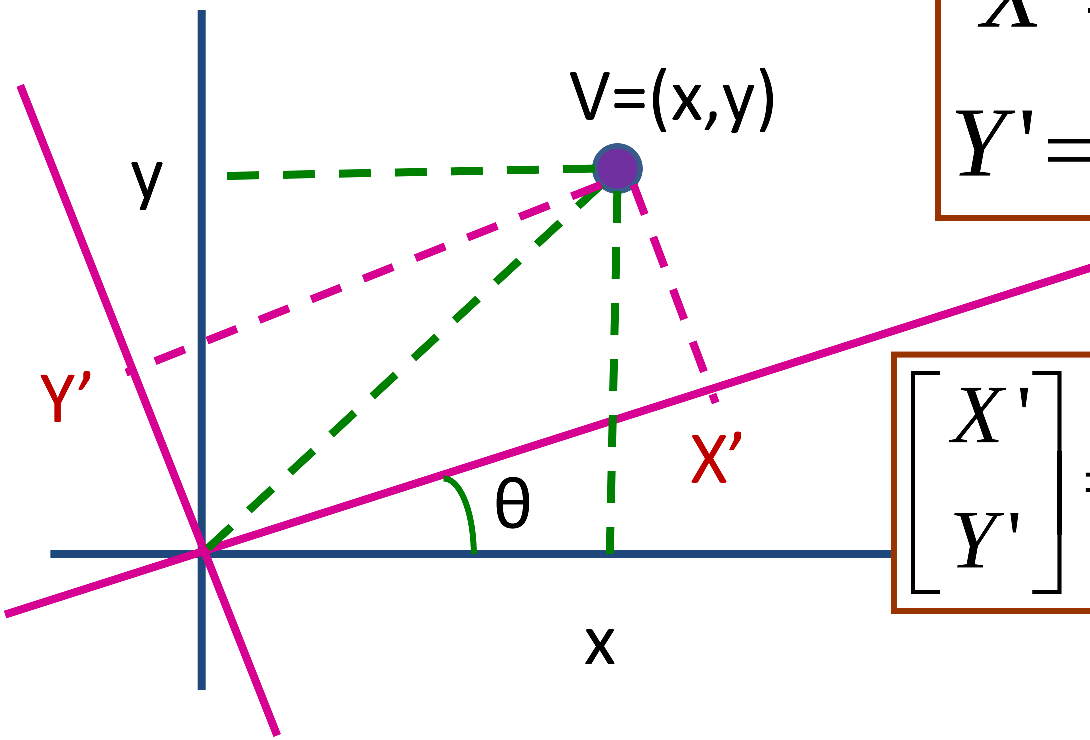
represent Y for a number of different input values for x, z, w.

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} x_1 & z_1 & w_1^2 & 1 \\ x_2 & z_2 & w_2^2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ x_n & z_n & w_n^2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

Common format for finding parameters a,b,c,d from observed data.

# Uses of matrix multiplication

A simple example involves the change of coordinates from one set to another – this is commonly needed for robotics and mechanics.




$$X' = x\cos\theta + y\sin\theta$$

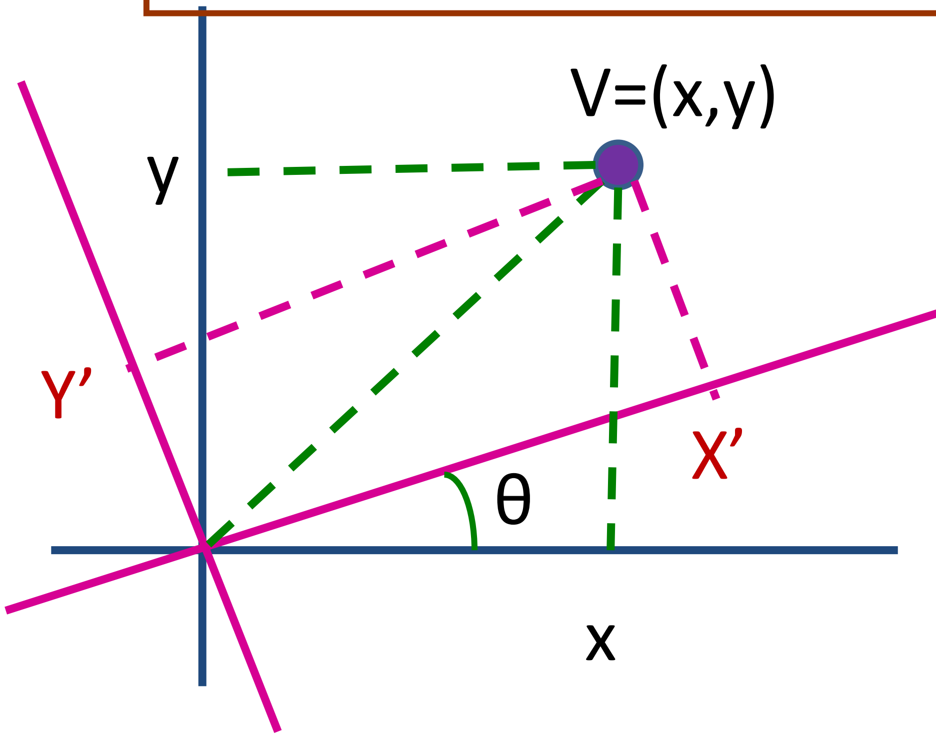
$$Y' = -x\sin\theta + y\cos\theta$$

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

# Change of coordinates

The matrix used to change from one coordinate frame to another may sometimes be denoted as a rotation matrix.

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = T(\theta) \begin{bmatrix} x \\ y \end{bmatrix}$$




Here T(θ) represents an anti-clockwise rotation of the frame of reference (or a clockwise rotation of the point).

# Rotation matrices

A pure rotation matrix has some key properties best discussed after videos on determinants.

The rotation must not change the size of the vector so:

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} = T(\theta)\begin{bmatrix} x \\ y \end{bmatrix} \quad \Rightarrow \quad \sqrt{x^2 + y^2} = \sqrt{X^2 + Y^2}$$

Moreover:

$$T(\theta)T(\theta)^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

# Magnification/scaling matrix

A diagonal matrix can be used to scale elements in particular axis directions.

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \eta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = D \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda x \\ \eta y \end{bmatrix}$$

Within robotics, one may need a transformation that uses both rotation and magnification, or indeed the magnification may be along axis which are not the standard basis set.

Such definitions are not given here, but can be obtained by combining transformations.

# Summary

Outlined some common uses for matrix multiplication.

- Representing and solving simultaneous equations.
- Representing data dependence on known parameters.
- Coordinate transformation and scaling.