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Matrices 9: determinants

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http://controleducation.group.shef.ac.uk/indexwebbook.html

http://www.shef.ac.uk/acse

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Introduction

- This video looks at the concepts of a determinant.
- This is useful for solving linear simultaneous equations and understanding properties of such systems of equations.

VIEWERS should note that a determinant is a **DEFINITION** – they cannot be proved or derived.

Also, a determinant is only defined for square matrices.

Determinant in brief

In conceptual terms, a determinant gives an indication of the ‘magnitude’ of a matrix, that is, on average, what scale of impact does it have on vectors when multiplying those vectors.

Students will notice that the effective magnification is direction dependent, and so the determinant is an average in some sense.

$$\begin{bmatrix}3 & 2\\ 2 & 3\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix}=\begin{bmatrix}5\\ 5\end{bmatrix};$$

Scaled by factor of 5.

Scaled by factor of 1.

Determinant – key fact

If the determinant is very small, then there exists a least one direction, which when multiplied by the matrix, results in a very small scaling factor.

$$\begin{bmatrix} 10 & 1 \\ 18 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 11 \\ 20 \end{bmatrix}; \quad \begin{bmatrix} 10 & 1 \\ 18 & 2 \end{bmatrix}\begin{bmatrix} 0.1 \\ -0.9 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix};$$

Scaled by factor of 16 (approx).

Scaled by factor of 0.1 (approx).

A zero determinant means at least one direction can be mapped to zero.

Determinant definition for 2 by 2 matrices

The determinant is defined as product of the diagonal elements minus the product of the off-diagonal elements:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}; \quad |A| = ad - bc$$

Note use of vertical lines around matrix is notation used to define determinant.

Try this on some examples.

Determinant computations

$$A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}; \quad |A| =$$

$$B = \begin{bmatrix} 10 & 1 \\ 18 & 2 \end{bmatrix}; \quad |B| =$$

Find the determinant of matrix D and the vectors w, z.

$$D = \begin{bmatrix} -1 & 4 \\ -5 & 8 \end{bmatrix}$$

$$\begin{vmatrix} -1 & 4 \\ -5 & 8 \end{vmatrix} =$$

$$w = \begin{bmatrix} -1 & 4 \\ -5 & 8 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \end{bmatrix} =$$

$$z = \begin{bmatrix} -1 & 4 \\ -5 & 8 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \end{bmatrix} =$$

Find the determinant of matrix D and the vectors w, z.

$$|D| = \begin{vmatrix} -0.25 & 1 \\ 0.5 & -2.25 \end{vmatrix} =$$

$$w = \begin{bmatrix} -0.25 & 1 \\ 0.5 & -2.25 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \end{bmatrix} =$$

$$z = \begin{bmatrix} -0.25 & 1 \\ 0.5 & -2.25 \end{bmatrix} \begin{bmatrix} -0.5 \\ 1 \end{bmatrix} =$$

Find the determinant of matrix F and vectors w, z

$$F = \begin{bmatrix} -1 & 3 \\ 4 & 12 \end{bmatrix} \qquad \begin{vmatrix} -1 & 3 \\ 4 & 12 \end{vmatrix} =$$

$$w = \begin{bmatrix} -1 & 3 \\ 4 & 12 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \end{bmatrix} =$$

$$z = \begin{bmatrix} -1 & 3 \\ 4 & 12 \end{bmatrix} \begin{bmatrix} 1 \\ -4 \end{bmatrix} =$$

Find x which makes the determinant zero.

$$M = \begin{bmatrix} 3-x & 10.4 \\ 5 & 12-x \end{bmatrix}$$

$$\begin{vmatrix} 3-x & 10.4 \\ 5 & 12-x \end{vmatrix} = (3-x)(12-x) - 52 = x^2 - 15x - 16$$

$$x^2 - 15x - 16 = (x-16)(x+1)$$

Summary

- Defined determinant for 2 by 2 matrices.
- Shown that the determinant can be zero even when the matrix has non-zero elements.
- Shown that a small determinant for matrix A indicates the existence of a direction x such that Ax is small.
- Shown that a zero determinant for matrix A indicates the existence of a direction x such that Ax =0.