Modelling and control summaries

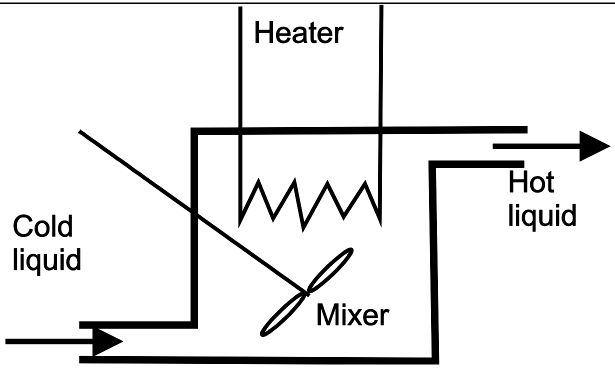
by Anthony Rossiter

1st order modelling 11: heat exchanger

This note looks at a simple heat exchanger with mass flow rate ρF, inlet temperature T_{in} and outlet temperature T and heating W. The aim is to model how the outlet temperature depends upon the inlet temperature and the supplied heating, the flow rate and the tank volume.

ASSUMPTIONS

1. For simplicity here we assume that the flow in (F) and the flow out are equal. This means the volume V in the tank is fixed.
2. The tank is well mixed so the temperature T throughout is equal and matches the outlet temperature.
3. The specific heat C_p and density ρ of the liquid are known (usually water).
4. Often heat is supplied by condensing steam. It is assumed that the heat supply can be approximated closely enough by just the latent heat λ of the steam.



MODELLING is done using an energy balance. To be more precise, we balance the rate of change of energy so it is a power balance.

ENERGY/POWER BALANCE

Accumulation of Energy = Energy in – Energy out

- The rate of energy coming into the tank is determined by inlet mass flow rate Fρ and inlet temperature. Energy flow rate in = $F\rho C_p T_{in}$
- The rate of energy coming out of the tank is determined by outlet mass flow rate Fρ and outlet temperature. Energy flow rate out = $F\rho C_p T$
- The rate of heat supply from condensing steam is given to the mass flow rate Q of steam as W=λQ
- The rate of change of energy stored in the tank comes from the difference between power in and power out from the above 3 items and hence:

$$\rho V C_p \frac{dT}{dt} = \rho F C_p (T_{in} - T) + \lambda Q$$

This can be rearranged into a standard 1st order model with two inputs:

1. Inlet temperature T_{in}
2. Mass flow rate of steam Q.

$$\frac{V}{F}\frac{dT}{dt} + T = T_{in} + \frac{\lambda}{\rho F C_p} Q$$

As the model is linear, superposition can be used to investigate the impact of changes in T_{in} and Q separately.

Using deviation variables

1. Deviation variables are sometimes useful and were discussed in the resource on mixing tanks. Here there are used without introduction.
2. The idea is to define states **relative to a known steady-state** (for example degrees Celsius is relative to the freezing point of water, altitude is relative to ground level on the earth, etc.).

STEP 1: Let a known operating point (at steady-state) have a given inlet temperature $T_{in,s}$ and steam flow Q_s. Note, at steady-state the derivative is zero.	$\frac{V}{F}\frac{dT}{dt} = T_i + (\frac{\lambda}{\rho F C_p}) Q - T = 0$ $\Downarrow$ $T_s = T_{i,s} + (\frac{\lambda}{\rho F C_p}) Q_s$
STEP 2: Define the deviation variables as deviations from the selected steady-state.	$T = T' + T_s; \qquad Q = Q' + Q_s$ $T_{in} = T_{in}' + T_{in,s};$
STEP 3: Substitute expressions above into the model.	$\frac{V}{F}\frac{d(T'+T_s)}{dt} + T' + T_s = T_i' + T_{i,s} + (\frac{\lambda}{\rho F C_p})(Q' + Q_s)$
STEP 4: Use the observations of STEP 1 to remove redundant terms.	$\frac{V}{F}\frac{d(T')}{dt} + T' = T_i' + (\frac{\lambda}{\rho F C_p}) Q'$
REMARK: In this example, because the underlying model is linear, superposition holds. Consequently the model with deviation variables matches the model with the original variables.	

Numerical example

Derive the model for an exchanger with the following parameters and hence sketch the response for a unit step change in input temperature and a unit step change in steam flow.

$V=3m^3$, $F=0.2m^3/s$, $\lambda=2.3\times10^6$ J/kg , $\rho=1000kgm^{-3}$, $C_p=4200$J/degree, $T_{i,s}=20$ degrees, $Q_s=10$kg/s

ANSWER:

1. The deviation model is given in step 4 above. Substitute in the parameters provided to give:

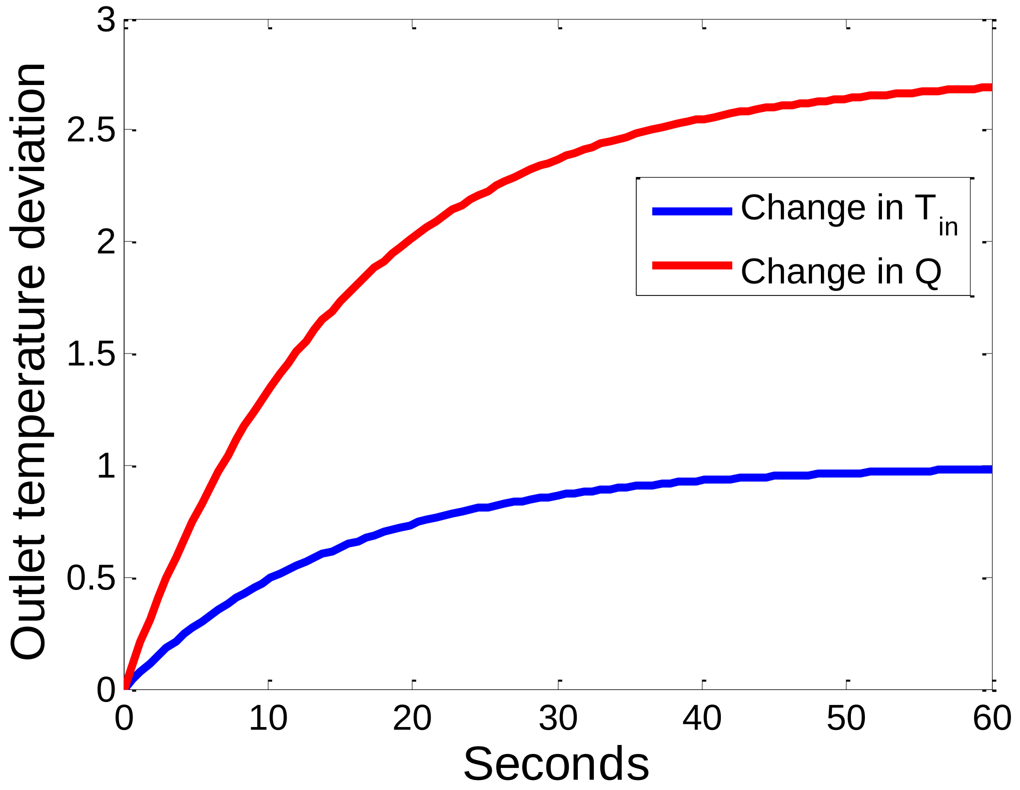
$$\frac{V}{F} = \frac{3}{0.2} = 15; \qquad \frac{\lambda}{\rho F C_p} = \frac{2.3\times10^6}{10^3\times0.2\times4200} = \frac{2.3}{0.2\times4.2} = 2.74$$

$$T_s = T_{i,s} + (\frac{\lambda}{\rho F C_p}) Q_s \quad \Rightarrow \quad T_s = 20 + 10\times2.74 = 47.4$$

2. Substitute in to form the 1st order model in time constant form as follows:

$$15\frac{d(T')}{dt} + T' = T_i' + 2.74\,Q'$$

3. It is clear therefore that a unit change in inlet temperature gives a unit steady-state change in outlet temperature and a unit change in steam flow gives a 2.74 degree uplift in outlet temperature. In both cases the time constant is 15, so the settling time is around 45-60sec.



NOTE: The actual temperature can be determined by adding the above to the steady-state T_s.