Modelling and control summaries

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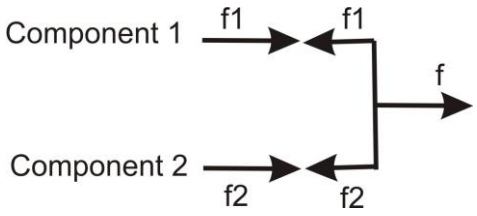
1st order modelling 2: spring-damper

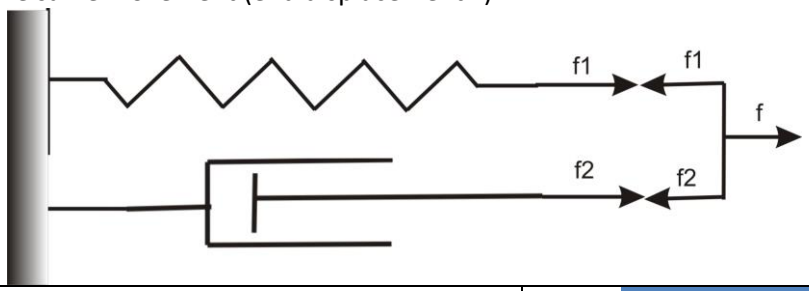
This summary assumes students are familiar with the basic modelling of springs and dampers, Newton's laws, concepts of force balance and also continuity (points joined together move the same distance).

FORCE BALANCE: The normal way to model spring-damper scenarios is to consider the force as being made up of different (parallel) components, here f1 and f2.

- f2 represents the friction force/drag. This is assumed to be proportional to speed!
- f1 represents the force required to extend the spring by 'x'.



Such a technique can be applied when the damper and spring are arranged in parallel and therefore, implicitly share the same movement (end displacement x).



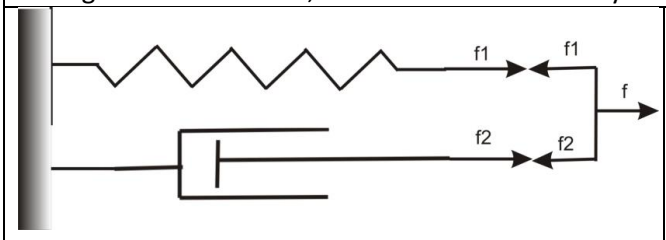
Using standard component equations of

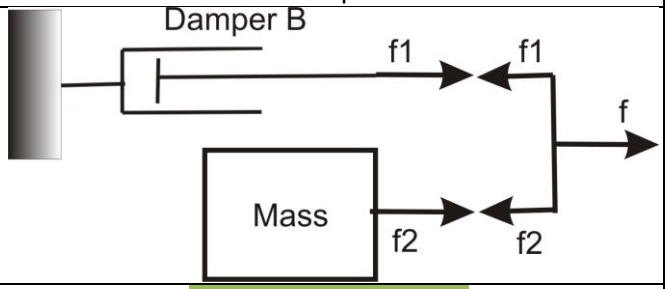
$$f2 = B\frac{dx}{dt}; \quad f1 = kx; \quad f = f1 + f2$$

$$f = kx + B\frac{dx}{dt}$$

The simple model for a spring-damper system is a **1st order ODE** with parameters the friction coefficient B and the spring stiffness k.

ANALOGIES: parallel mass-damper and parallel spring-damper systems have analogous models in that both give 1st order ODEs, one with state of velocity and the other with a state of displacement.

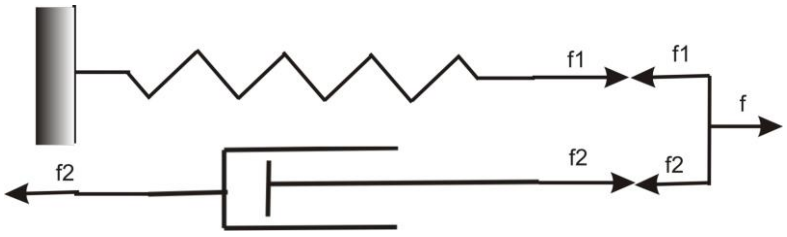
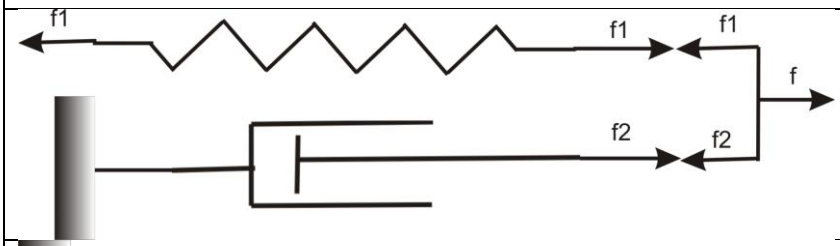
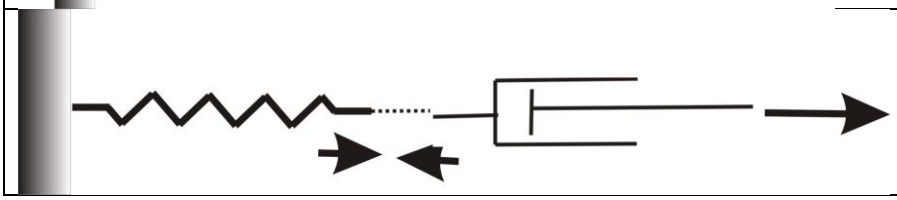




$$f = kx + B\frac{dx}{dt}$$

$$f = Bv + M\frac{dv}{dt}$$

REMARKS: Other arrangements of springs and dampers will exist and reader needs to be careful to write down the associated displacements, force balance, component equations and so forth carefully as slightly different models will arise with each arrangement.

	ALMOST PARALLEL, but note the damper does not share the same displacement as both ends are free. $\left.\begin{aligned} f1 &= kx \\ f2 &= B(\frac{dx}{dt} - \frac{dy}{dt}) \\ f &= f1 + f2 \end{aligned}\right\} \Rightarrow$
	ALMOST PARALLEL, but note the spring does not share the same displacement as both ends are free.
	SERIES arrangement as the force is shared but the displacements/extensions are different.