Modelling and control summaries

by Anthony Rossiter

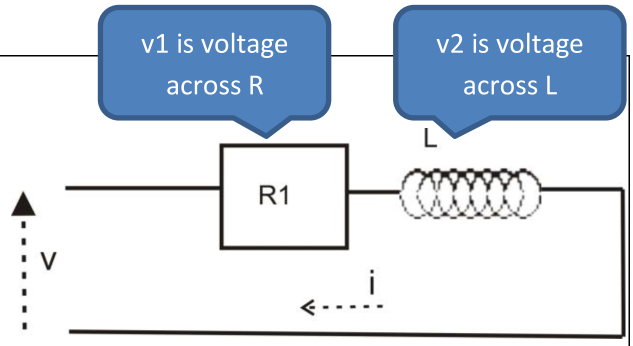
1^{st} order modelling 4: resistor-inductor

This summary assumes students are familiar with the basic modelling equations for electrical components and also the definitions of Kirchhoff's current and voltage laws. 1^{st} order models of simple electrical circuits tend to arise for series arrangements of components and thus that is the focus here. For series arrangements we use Kirchhoff's voltage laws (KVL).

Consider a series arrangement with a single resistor and a single inductor. To model this scenario 3 equations are needed.

1. Equations to represent each component.
2. Application of KVL.

REMARK: Readers are advised to begin by constructing a full labelled figure, as here; include all voltages and currents.



$$v1 = iR$$

v1 is voltage across R

$$v2 = L\frac{di}{dt}$$

v2 is voltage across L

$$v = v1 + v2$$

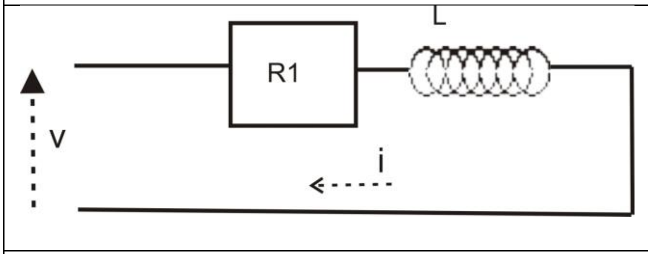
KVL

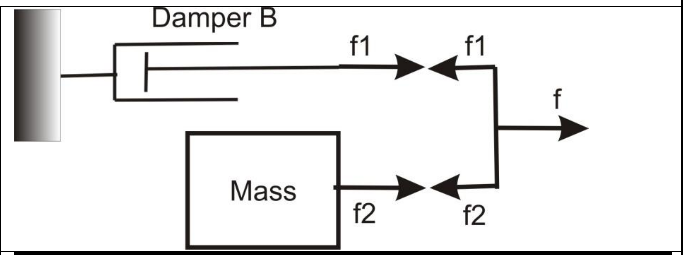
HENCE:

$$v = iR + L\frac{di}{dt}$$

The simple model for a resistor-inductor system is a **1^{st} order ODE** with parameters the resistance R and the inductance L.

ANALOGIES: A series RL circuit is often taken to be analogous to a parallel mass-damper system.





$$v = iR + L\frac{di}{dt} \quad \equiv \quad \frac{1}{R}v = i + \frac{L}{R}\frac{di}{dt}$$

$$f = Bv + M\frac{dv}{dt} \quad \equiv \quad \frac{1}{B}f = v + \frac{M}{B}\frac{dx}{dt}$$

It is clear that both give 1st order ODEs, but what other analogies are there? Such insights can be useful when tackling more complex scenarios. It is clear that:

1. Model gain depends only on the resistance and damper and thus these two have an analogous role.
2. The inductance and mass affect the time constant as a multiplying factor – again an analogous impact.
3. Current i is analogous to velocity v (states in the ODEs). [Also both have units per sec]

1. An inductor stores energy link to flow rate i and a resistor dissipates energy as heat.

2. A mass stores energy linked to velocity v and a damper dissipates energy as heat.

3. Voltage is analogous to force (distributed between the two components).

From these statements it is clear that the components have analogous behaviours and moreover the modelling steps are based on an analogous concept (KVL and force balance).

GENERAL ANALOGIES

Voltage with Force.

Displacement with charge.

Damper with resistor.

Spring with capacitor.

Current with velocity.

Parallel mechanical with series electrical.