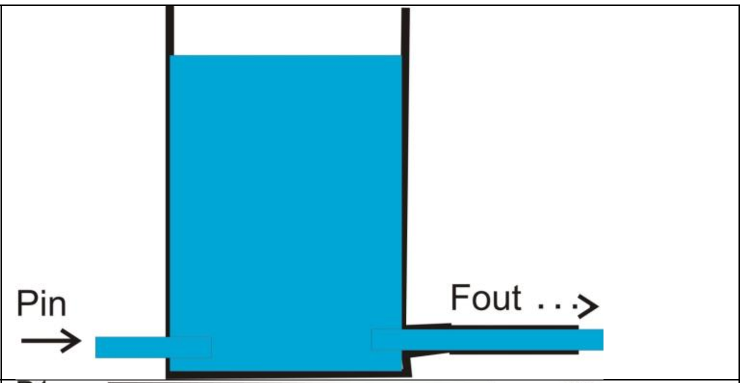
# Modelling and control summaries

by Anthony Rossiter

## 1st order modelling 5: fluid systems

**This note looks at a simple tank system and shows that it has analogies with a resistor-capacitor circuit.**

Consider a tank which has fluid flowing in (tank input) and fluid flowing out.
Define the state as the depth in the tank and find a model for depth h in terms of the tank parameters and the in flow.
A is the cross-sectional area so that volume stored = V =Ah



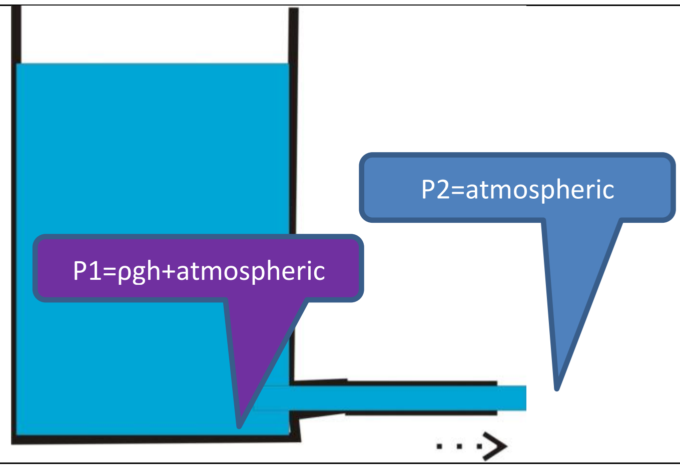

**ASSUMPTION**: The flow through a restriction is proportional to the pressure difference across the restriction.
In the case, the restriction is at the plug hole or outlet pipe. Assume that the model for flow is:


$$R(P1 - P2) = F$$

The difference between the pressure at the bottom of the tank and the pressure at the outlet (assumed atmospheric) is taken to be density*gravity*depth = ρgh.

Hence, for appropriate constant R (essentially a conductance term):

$$R(\rho gh) = F_{out}$$




**CHANGE OF DEPTH**:
A change in the depth in the tank means a change in volume of liquid stored.
One can write a volume balance by inspection based on the difference between flow into the tank and flow out.

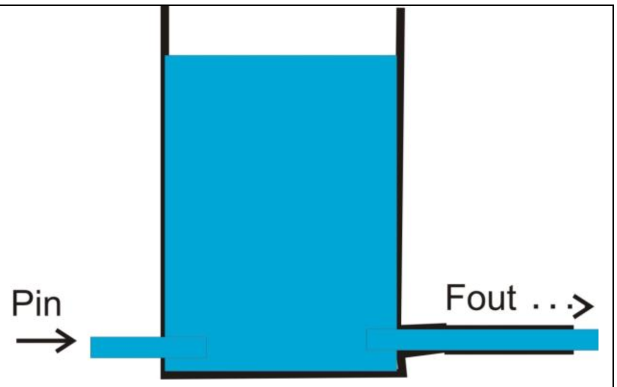
$$\frac{dV}{dt} = A\frac{dh}{dt} = F_{in} - F_{out}$$

Now consider that an in flow is supplied by a high pressure inlet, with a different resistance to flow, so that:

$$R_{in}(P_{in} - \rho gh) = F_{in}$$

Combining these three equations together one finds.

$$A\frac{dh}{dt} = R_{in}(P_{in} - \rho gh) - R\rho ghF_{out}$$




Rearranging into standard time constant form:

$$A\frac{dh}{dt} + (R_{in} + R)\rho gh = R_{in}P_{in} \quad \equiv \quad \frac{A}{(R_{in} + R)\rho g}\frac{dh}{dt} + h = \frac{R_{in}}{(R_{in} + R)\rho g}P_{in}$$

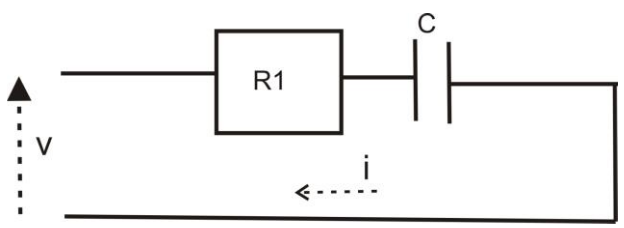
The time constant increases with the cross-section area and reduces if the overall conductance ($R_{in}$+R) increases. The gain depends upon the ratio of the inlet and outlet conductances. The 'input' is the pressure (above atmospheric) of the inlet pipe.

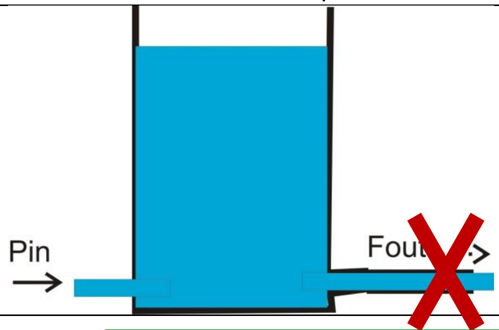
**REMARK**: if the outlet pipe is closed, this model reduces to:

$$\frac{A}{R_{in}\rho g}\frac{dh}{dt} + h = \frac{1}{\rho g}P_{in}$$

NOTE, steady-depth no longer linked to $R_{in}$!

**ANALOGIES**: A tank fed by a high pressure inlet and with no outlet is analogous to a resistor capacitor circuit. The analogy is clearer using tank volume as the state rather than tank depth where V=Ah.




$$Cv = q + CR1\frac{dq}{dt}$$

$$\frac{A}{R_{in}\rho g}\frac{dV}{dt} + V = \frac{A}{\rho g}P_{in}$$

1. Model gain depends on the capacitance or tank cross-sectional area (this is equivalent to capacitance in that it relates to the quantity stored per unit input – charge or volume).
2. The time constant is linked to capacitance and resistance [CR1] or equivalently conductance and cross-sectional area [A/$R_{in}$].

**REMARK**: Interested readers may like to investigate the links between the tank system with the outflow open and an equivalent electrical circuit.