Modelling and control summaries

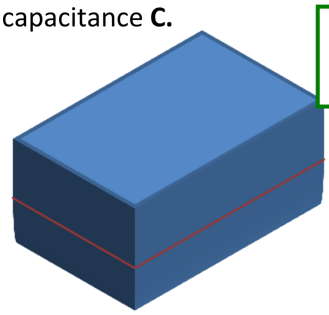
by Anthony Rossiter

1st order modelling 6: heating system

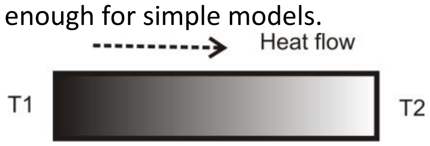
This note looks at a simple heating system which as thermal capacitance (stores energy in proportion to temperature) and heat losses in proportion to the temperature difference with the surroundings.

Let energy stored by denoted as **Q** and temperature as **T**, then with thermal capacitance **C.**

$$Q = CT$$

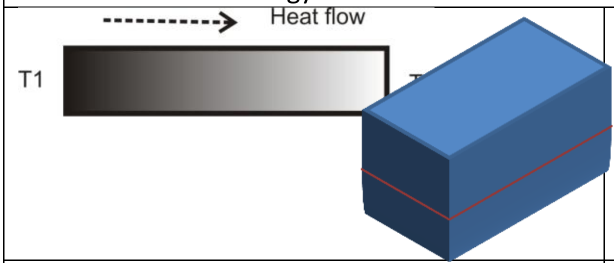


Assume that heat flow W can be represented as follows – this is close enough for simple models.



$$(T1 - T2) = K_H W$$

Now form a system which has elements of both, that is thermal capacitance to store energy as heat and poor insulation so that energy is loss whenever there is a temperature difference.



$$\left.\begin{array}{l} Q = CT_2 \\ K_H \dfrac{dQ}{dt} = T_1 - T_2 \end{array}\right\} \Rightarrow \quad K_H C \frac{dT_2}{dt} = T_1 - T_2$$

This is clearly a first order model where the driving input is the external temperature. Gain is unity and the time constant depends upon the effectiveness of the insulation and the thermal capacitance.

$$K_H C \frac{dT_2}{dt} + T_2 = T_1$$

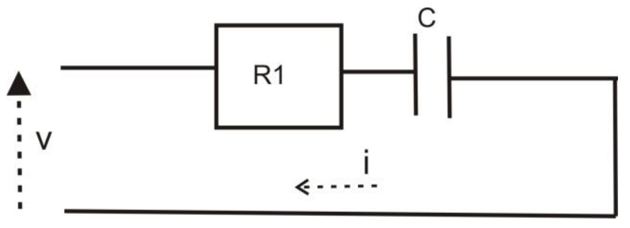
Next consider what would happen if some **internal heating H is added**, so for example this system could represent a simple house which looses heat to the outside through the walls and windows but has radiators/fires supplying heat.

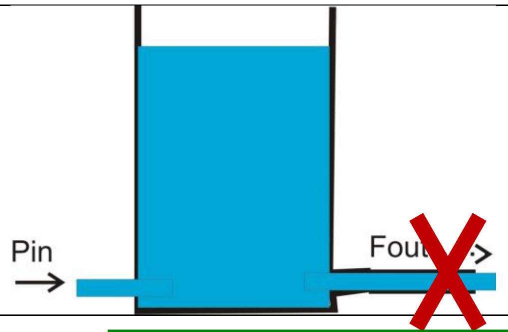
$$\frac{dQ}{dt} = C\frac{dT_2}{dt} = \frac{(T_1 - T_2)}{K_h} + H$$

This is equivalent to a 1^{st} order model with 2 inputs (different associated gains), these being external temperature which you cannot choose and internal heating which you can.

$$CK_h \frac{dT_2}{dt} + T_2 = T_1 + K_h H$$

ANALOGIES: The heating system is analogous to the resistor-capacitor circuit, the tank fed by a high pressure inlet and a spring damper system.



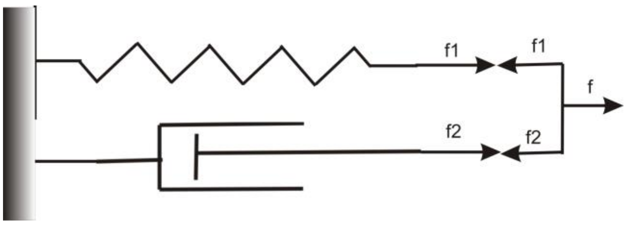


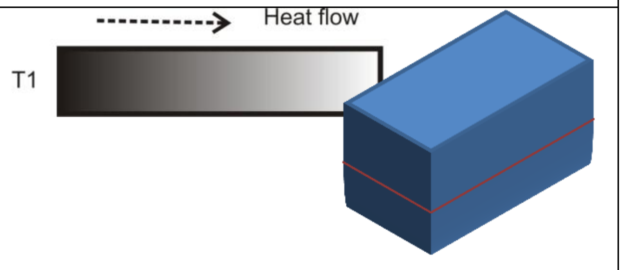
$$Cv = q + CR1\frac{dq}{dt}$$

$$\frac{A}{R_{in}\rho g}\frac{dV}{dt} + V = \frac{A}{\rho g}P_{in}$$

$$\frac{B}{k}\frac{dx}{dt} + x = \frac{1}{k}f$$

$$CK_h\frac{dT_2}{dt} + T_2 = T_1 + K_h H$$





1. Model gain depends on the capacitance or tank cross-sectional area or spring stiffness (resistance to change of state) or thermal resistance/conductivity.
2. The time constant is linked to capacitance and resistance [CR1] or equivalently conductance and cross-sectional area [A/R_{in}] or damping/stiffness (B/k) or thermal capacitance and thermal resistance (CK_h).

Role of capacitor (**stores charge**) is analogous to tank cross-sectional area (**stores volume**) or spring (**stores displacement**) or thermal capacitance (**stores energy**).
Role of resistor (**resists current flow**) is analogous to the inlet pipe resistance (**resists fluid flow**) or the damper (**resists velocity**) or the insulation (**resists heat flow**).