Modelling and control summaries

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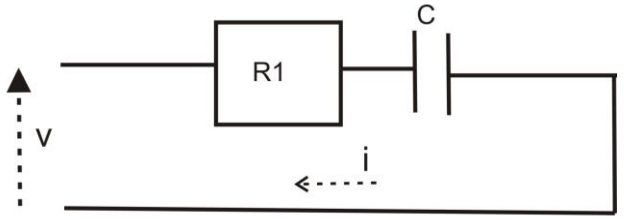
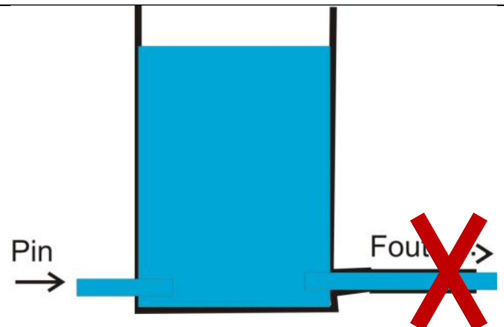
1^{st} order modelling 7: time constant form

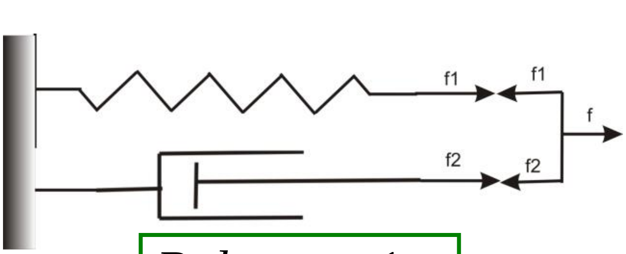
This note assumes readers have access to 1^{st} order modelling 1-6 and takes the models below ad the associated notation from those as given.
The purpose of this note is to introduce the TIME CONSTANT FORM and use these 6 systems as examples.



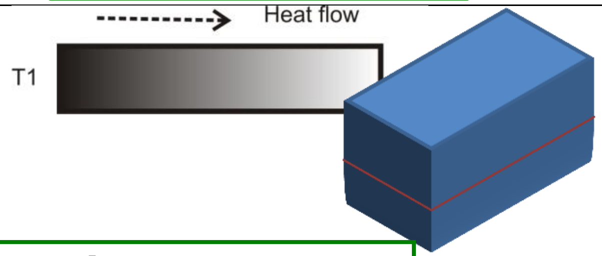
$$Cv = q + CR1\frac{dq}{dt}$$



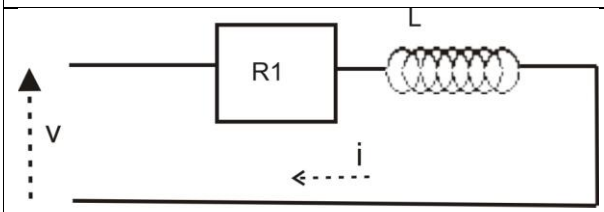
$$\frac{A}{R_{in}\rho g}\frac{dV}{dt} + V = \frac{A}{\rho g}P_{in}$$



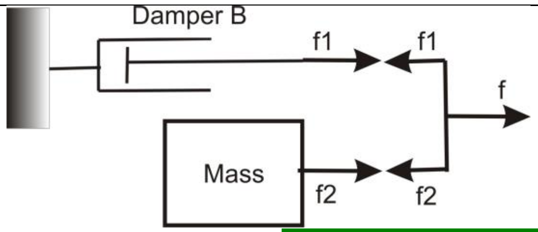
$$\frac{B}{k}\frac{dx}{dt} + x = \frac{1}{k}f$$



$$CK_h\frac{dT_2}{dt} + T_2 = T_1 + K_h H$$



$$L\frac{di}{dt} + iR1 = v$$



$$f = M\frac{dv}{dt} + Bv$$

TIME CONSTANT FORM: This applies to simple 1st order ODEs. The idea is that the coefficients are structured in a very particular way so that the coefficient of the state is always unity.

1. The coefficient on the derivative is the time constant.
2. The coefficient on the input is the steady-state gain (often denoted gain for shorthand).

Arbitrary ODE with non-zero coefficients, state x and input u.	In time constant form, the coefficient of the 'x' term is set to unity and hence:
$$a\frac{dx}{dt}+bx=cu$$	$$\left(\frac{a}{b}\right)\frac{dx}{dt}+x=\left(\frac{c}{b}\right)u \quad \equiv \quad T\frac{dx}{dt}+x=Ku$$ Time constant T; Coefficient=1; Gain K

Some examples are given next.

Readers may like to think about what analogies they can draw between these systems in that the behaviours are linked directly to T and K.

1. T dictates the settling time of the system (or speed of response).
2. K dictates the steady-state ration of output (or state) to input.

$$L\frac{di}{dt}+iR1=v \quad \Rightarrow \quad \frac{L}{R1}\frac{di}{dt}+i=\frac{1}{R1}v \quad \Rightarrow \quad T=\frac{L}{R1}, \quad K=\frac{1}{R1}$$

$$f=M\frac{dv}{dt}+Bv \quad \Rightarrow \quad \frac{M}{B}\frac{dv}{dt}+v=\frac{1}{B}f \quad \Rightarrow \quad T=\frac{M}{B}, \quad K=\frac{1}{B}$$

$$CK_h\frac{dT_2}{dt}+T_2=T_1 \quad \Rightarrow \quad T=CK_h, \quad K=1$$

$$\frac{B}{k}\frac{dx}{dt}+x=\frac{1}{k}f \quad \Rightarrow \quad T=\frac{B}{k}, \quad K=\frac{1}{k}$$

$$\frac{A}{R_{in}\rho g}\frac{dV}{dt}+V=\frac{A}{\rho g}P_{in} \quad \Rightarrow \quad T=\frac{A}{R_{in}\rho g}, \quad K=\frac{A}{\rho g}$$

$$R\frac{dq}{dt}+\frac{q}{C}=v \quad \Rightarrow \quad RC\frac{dq}{dt}+q=Cv \quad \Rightarrow \quad T=RC, \quad K=C$$