Modelling and control summaries

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1^{st} order modelling 8: tank level system

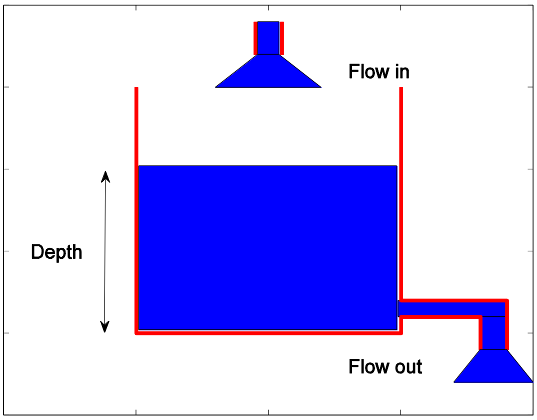
This note looks at a simple tank system which has flow in (the input) and a flow out (from the bottom of the tank through a restriction). The output or state of interest is the depth 'h' in the tank.

The aim here is to form a model for the depth in the tank and in particular determine how this depends upon both the flow in and the flow out.

An assumption is made that the flow through a restriction is proportional to the pressure difference across the restriction and thus can be modelled with an equation of the form:

$$f = K(P_1 - P_2)$$

Where P_1,P_2 are the pressures at each side.



1a) A simple volume balance can be used to give the basic model, or more specifically, the rate of change of volume in the tank depends on the difference between the flow in and the flow out. Hence:

$$\boxed{\frac{dV}{dt} = f_{in} - f_{out}}$$

1b) For convenience, assume the tank has a constant cross section area 'A' so that V=Ah.
Therefore one can rewrite the volume balance as follows:

$$\boxed{A\frac{dh}{dt} = f_{in} - f_{out}}$$

2a) The pressure difference between the bottom of the tank and the outlet is assumed to be solely due to the pressure of the water in the tank.
It is well known that this is given by the formulae: P=ρgh (where ρ is density of water and g acceleration due to gravity).

2b) The f_{out} can now be represented by a simple equation

$$\boxed{f_{out} = K\rho gh}$$

Where K is a constant which depends upon the outlet (cross-sectional area, length, material, etc) and is non trivial to model from 1^{st} principles.

3) Combining the two equations from 1b,2b above gives a first order model for the tank system.

$$\boxed{\left\{A\frac{dh}{dt} = f_{in} - K\rho gh\right\} \equiv \left\{\underbrace{\left(\frac{A}{K\rho g}\right)}_{T}\frac{dh}{dt} + h = \underbrace{\left(\frac{1}{K\rho g}\right)}_{Gain} f_{in}\right\}}$$

- Time constant T depends on resistance of outlet pipe and cross-sectional area.
- Gain depends on resistance of outlet pipe but NOT the cross sectional area.

If cross-sectional area is increased, the time constant increases.

If resistance to flow (1/K) is increased, time constant increases and steady-state gain increases.