Modelling and control summaries

by Anthony Rossiter

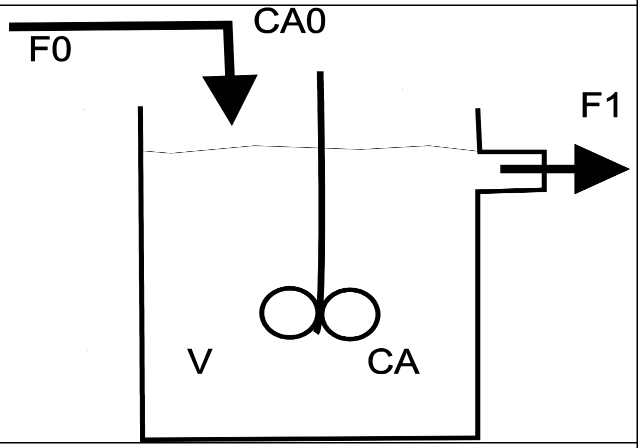
1st order modelling 9: mixing tank

This note looks at a simple mixing tank system which has flow in at a concentration C_{A0} and flow out a different concentration C_A. The aim is to model how the outlet concentration depends upon the inlet concentration, flow rates and tank volume.

ASSUMPTIONS

1. For simplicity here we assume that the flow in F0 and the flow out F1 are equal. This means the volume V in the tank is fixed.
2. The tank is well mixed so the concentration in the tank matches the outlet concentration.
3. The solvent and product A have the same density.
4. There is no reaction in the tank involving A.

MODELLING is done using a **mass or molar balance** (these are equivalent). To be more precise, we balance the rate of change of mass A within the tank.



MASS BALANCE

Accumulation of A = A (input) – A (output) + generation of A – expenditure of A

Note that the assumption here is that generation and expenditure (say from reactions) are zero.
DEFINE M_A as mass per mole for pure A, so mass of A per m^3 within solvent is C_AM_A

- The rate of A coming into the tank is determined by inlet flow F0 and inlet concentration.
  Mass flow rate in = $F_0C_{A0}\,M_A$
- The rate of A leaving the tank is determined by outlet flow f1 and outlet concentration.
  Mass flow rate out = $F_1C_A\,M_A$
- The total mass in the tank= VC_AM_A so, given V and M_A are constant. Therefore the rate of change of mass in the tank is given by

$$VM_A\frac{dC_A}{dt}=(M_AF_0C_{A0}-M_AF_1C_A)$$

It is noted that the term M_A is a common factor throughout and so can be removed. Also, the assumption is that F_1=F_0=F and hence:

$$\left\{V\frac{dC_A}{dt}=(FC_{A0}-FC_A)\right\}\equiv\left\{\underbrace{\left(\frac{V}{F}\right)}_{T}\frac{dC_A}{dt}+C_A=\underbrace{1}_{gain}\times C_{A0}\right\}$$

1. Because there is no reaction, the steady-state gain is clearly one, that is the concentration in the tank tends to the concentration of the input flow.
2. The time constant increases if the volume increases and decreases if the flow increases. Both of these should be intuitively obvious.

Numerical example

Find the response of a mixing tank with the following data and subject to a step increase in C_{A0} of magnitude 0.925mole/m^3.

F0=0.085m^3/min; V=2.1m^3;$C_A(0)$=0.925 mole/m^3; C_{A0}=0.925mole/m^3

Assume the system is initially at steady state before the step change in C_{A0}.

ANSWER:

1. First note that the effective inlet concentration is now C_{A0}=1.85 as the data gives an increase from the current value.
2. The effective initial condition $C_A(0)$ for the outlet concentration is 0.925 as the system begins from steady-state.
3. In standard time constant form, the model and data are:

$$\left\{ \underbrace{\left(\frac{V}{F}\right)}_{T} \frac{dC_A}{dt} + C_A = \underbrace{1}_{gain} \times C_{A0} \right\}; \quad C_A(0) = 0.925, \quad C_{A0} = 1.85, \quad \frac{V}{F} = \frac{2.1}{0.085} = 24.7$$

Consequently, using standard responses for a 1st order model:

$$C_A(t) = 0.925 + 0.925(1 - e^{-\frac{t}{24.7}})$$

Summary

1. Derived model for a simple mixing tank, with and without deviation variables.
2. Model is linear and takes a standard 1st order format.
3. Time constant linked to tank volume and flow rate.
4. Steady-state gain is unity as there is no consumption or production.