Modelling and control summaries

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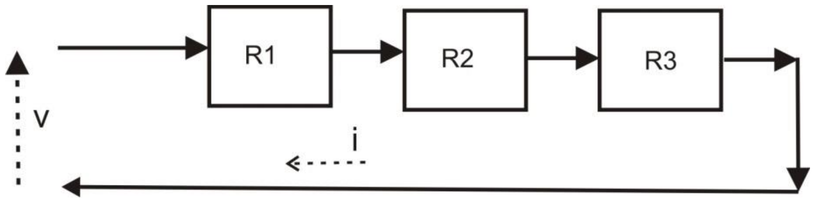
Modelling principles and analogies 1:

Modelling resistors in series

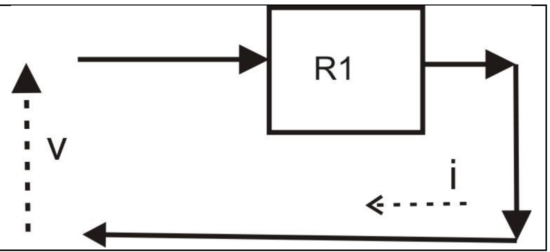
Consider an electrical circuit with supplied voltage *v(t)*, a set of resistors arranged in series (such as in figure here) and a current flow *i(t)*.

What is the relationship between the current, the voltage and the resistances?



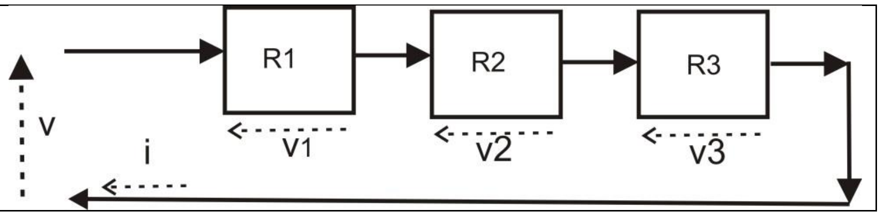
These notes will give a brief overview of the foundational assumptions required to answer this question.

Model for a single resistor

The current flow is proportional to the voltage. Specifically, the dependence can be expressed as: $$v = iR_1$$	

Kirchhoff's Voltage Law

All the voltages acting round a loop of a circuit must sum algebraically to zero. A*lgebraically* means attributing opposite signs to voltages acting in opposite directions, say, clockwise and counter clockwise respectively. Hence, for the general loop illustrated below:

$$v_1 + v_2 + v_3 = v$$	

Assumption: Electrical components in series share the same current.

Combining the rule for a single resistor with Kirchhoff's voltage law and the assumption, one can now derive a model for many resistors in series.

Model for a two resistors in series

Modelling simply requires a statement of each observation above and then a simplification of the resulting equations.

1. The voltage across resistor R1

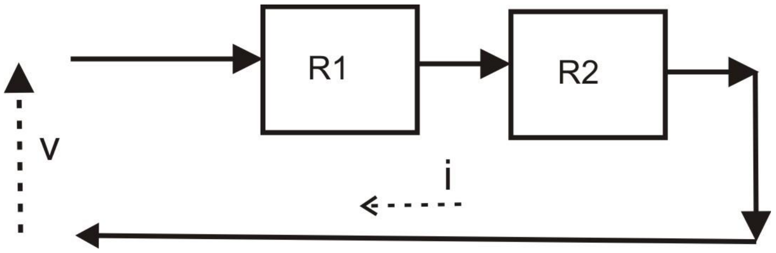
$$v_1 = iR_1$$

2. The voltage across resistor R2

$$v_2 = iR_2$$

3. Kirchhoff's voltage law:

$$v_1 + v_2 = v$$



Combining these three equations gives:

$$iR_1 + iR_2 = v \quad \Rightarrow \quad v = i(R_1 + R_2)$$

Model for a three resistors in series

Modelling requires a statement of each observation above and then a simplification of the resulting equations.

1. The voltage across resistor R1

$$v_1 = iR_1$$

2. The voltage across resistor R2

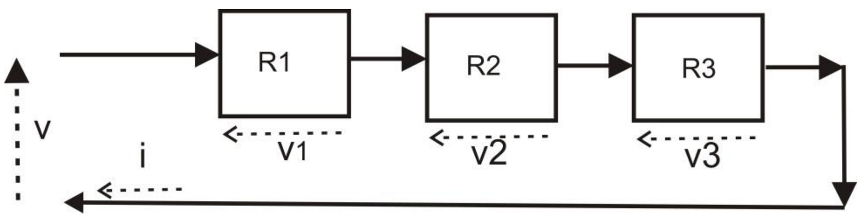
$$v_2 = iR_2$$

3. The voltage across resistor R3:

$$v_3 = iR_3$$

3. Kirchhoff's voltage law:

$$v_1 + v_2 + v_3 = v$$



Combining these four equations gives:

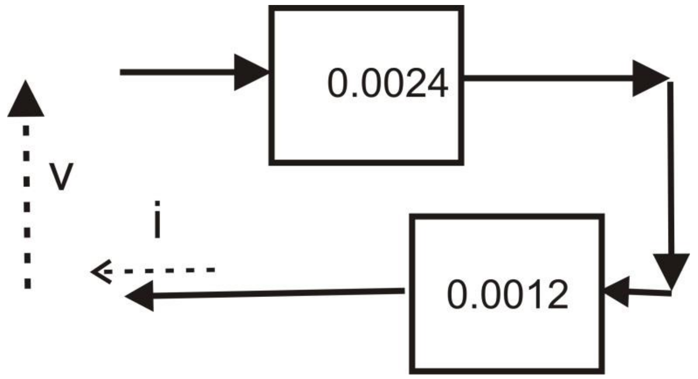
$$iR_1 + iR_2 + iR_3 = v \quad \Rightarrow \quad v = i(\underbrace{R_1 + R_2 + R_3}_{R_t})$$

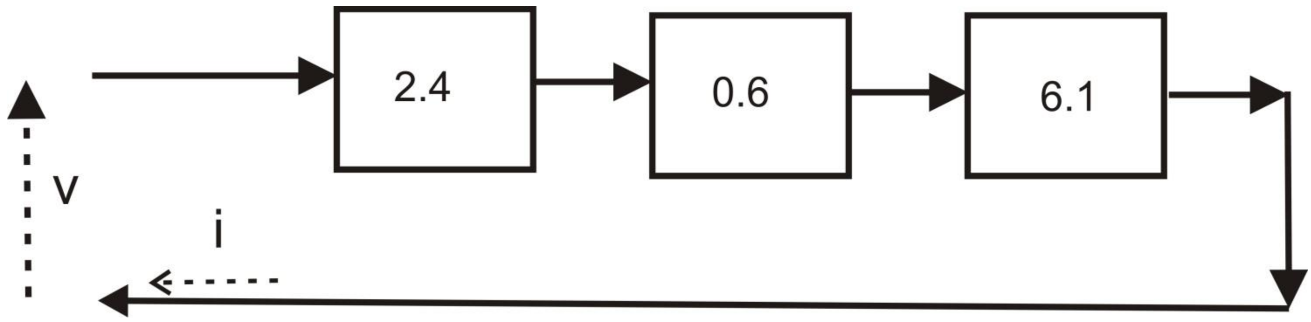
Remark: It is clear that for n resistors in series, the corresponding model will be:

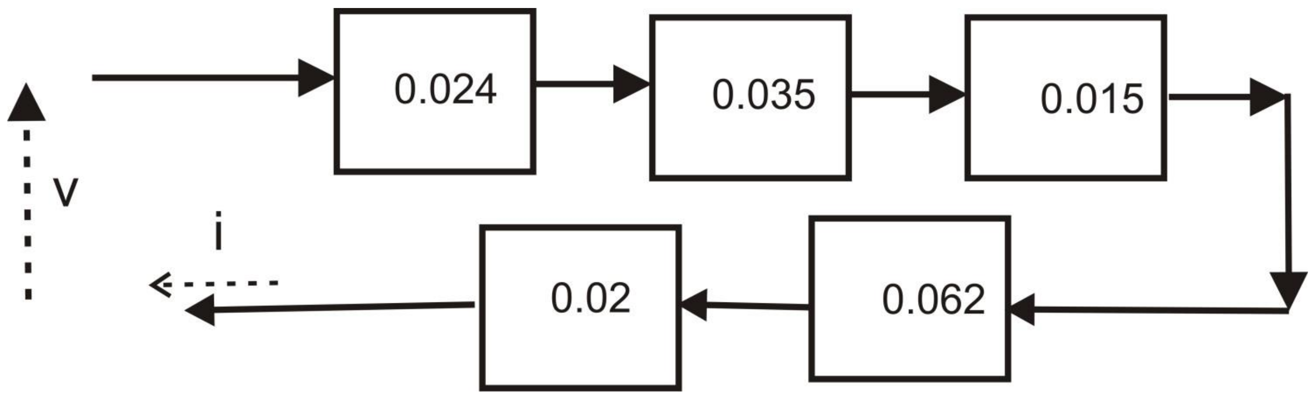
$$iR_1 + iR_2 + \cdots + iR_n = v \quad \Rightarrow \quad v = i(\underbrace{R_1 + R_2 + \cdots + R_n}_{R_t})$$

Tutorial questions

Find the overall circuit resistance for the following circuits. (Any numbers given represent the resistance in ohms).







ANSWERS: (i) 0.0036ohms (ii) 9.1ohms; (iii) 0.156ohms