Modelling and control summaries

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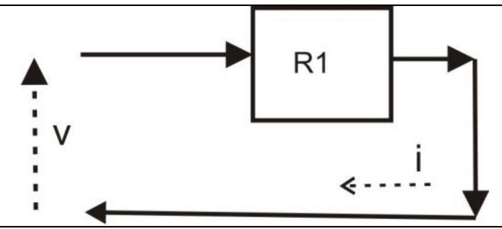
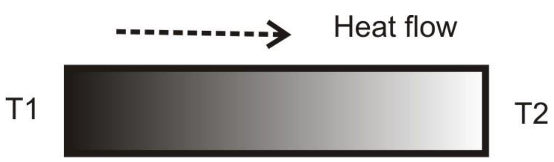
Modelling principles and analogies 3:

Heat conductors

The detailed derivations required here mirror those in the sheet on fluid flow and hence are not repeated and left to the reader who should open the fluid flow sheet and add extra rows in each table for heat conductors. For simplicity these slides assume no heat loss along the sides.

Model for a single resistor and analogy to model for a single pipe and heat conduction

The current flow is proportional to the voltage. Specifically, the dependence can be expressed as: $$v = iR_1$$	
The fluid flow F is proportional to the pressure difference, and hence: $$P1 - P2 = FR_1$$	
The heat flow W along a conductor depends upon the temperature difference between the two ends. $$(T1 - T2) = K_H W$$	

The analogies are obvious:

Temperature difference T1-T2 is analogous to pressure difference P1-P2 and to voltage v.	**Heat flow W is analogous to fluid flow F is analogous to current flow i.**	**Resistance to heat flow is analogous to electrical resistance and to pipe flow resistance.**

MANY pipes in series are analogous to many electrical resistors in series and many conductors in series

It is clear that a way of reducing heat flow is to make the path longer, or to add more conductors in series. For example, when you are cold you put on more layers (layers in series), or insulate the walls of your house (layers in series) and so forth. Hence, conductors in series have a higher resistance to heat flow!

Many pipes in series can be modelled by considering the pressure drop across each pipe in turn given equations as follows (for overall pressure drop of P1-P2):	$\left.\begin{matrix} P1-P3=FR_1 \\ P3-P4=FR_2 \\ P4-P2=FR_3 \end{matrix}\right\} \Rightarrow \quad P1-P2=F(R_1+R_2+R_3);$
Many conductors in series can be modelled by considering the temperature drop across each conductor in turn given equations as follows (for overall temperature drop of T1-T4):	$\left.\begin{matrix} T1-T2=WK_1 \\ T2-T3=WK_2 \\ T3-T4=WK_3 \end{matrix}\right\} \Rightarrow \quad T1-T4=W(K_1+K_2+K_3);$
Analogously, resistors in series considering the voltage drop across each resistor in turn:	$\left.\begin{matrix} v1=iR_1 \\ v2=iR_2 \\ v3=iR_3 \end{matrix}\right\} \Rightarrow \quad v=i(R_1+R_2+R_3);$

MANY pipes in parallel are analogous to many electrical resistors in parallel and many conductors in parallel

Using the analogies between the underlying models for single pipes, single resistors and conductors, this result is obvious. The more routes heat has to escape or travel through, the greater the heat loss! So, increasing the number of paths (such as increasing window area in a house) can increase the flow of heat (lower resistance to heat flow).

Resistors in parallel $\frac{v}{R_1}+\frac{v}{R_2}+\cdots+\frac{v}{R_n}=i_1+i_2+\cdots+i_n=i$ $\Rightarrow \quad v=i\underbrace{\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\cdots+\frac{1}{R_n}}\right)}_{R_t}$	Pipes in parallel $\frac{\delta P}{R_1}+\frac{\delta P}{R_2}+\cdots+\frac{\delta P}{R_n}=F_1+F_2+\cdots+F_n=F$ $\Rightarrow \quad \delta P=F\underbrace{\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\cdots+\frac{1}{R_n}}\right)}_{R_t}$
Conductors in parallel $\frac{\delta T}{K_1}+\frac{\delta T}{K_2}+\cdots+\frac{\delta T}{K_n}=W_1+W_2+\cdots+W_n=W$ $\Rightarrow \quad \delta T=W\underbrace{\left(\frac{1}{\frac{1}{K_1}+\frac{1}{K_2}+\cdots+\frac{1}{K_n}}\right)}_{K_t}$	$\delta P=P1-P2$ $\delta T=T1-T2$